

## A Modified Pre-Coding Technology for PAPR Reduction in V-OFDMsystem

<sup>1</sup>Bin Lei, <sup>2</sup>Honggui Deng, <sup>3</sup>Yu Jiang

Department of Electronic Science and Technology, Central South University,  
Lushan Road,410083 Changsha, China, +86-0731-88836443

\*Corresponding author, e-mail: denghonggui@163.com

Accepted on March 07, 2015

**Abstract:** The Vector Orthogonal Frequency Division Multiplexing (V-OFDM) system is a generalized version of the conventional Orthogonal Frequency Division Multiplexing (OFDM) system, and also has the disadvantage of high peak-to-average power ratio (PAPR). However, most technologies used in OFDM cannot be applied to a V-OFDM system directly because of existing difference between the OFDM and V-OFDM systems. After further research on the characteristics of V-OFDM system, a modifying pre-coding technology is proposed for PAPR reduction in V-OFDM system, where power of each vector block (VB) is assigned to V-OFDM block for PAPR reduction. The simulation results show that the modified pre-coding technology can reduce PAPR more effectively and has a 1 dB gain on signal noise ratio (SNR) when the bit error rate (BER) reached  $10^{-3}$  in the case of  $M = 4$  ( $M$  represents the length of VBs) in V-OFDM system. So the modified pre-coding technology is an attractive solution to the high PAPR of V-OFDM system.

**Keywords:** Vector Orthogonal Frequency Division Multiplexing (V-OFDM); modified pre-coding technology; PAPR.

### Introduction

Single antenna V-OFDM is a generalization of OFDM technology [1-2], where the traditional OFDM scalar channel and symbols are transformed into vector channel and OFDM VBs, and a certain number of cyclic prefix is inserted to each VB. Through the choice of system parameters, V-OFDM technology can make tradeoff between computational complexity and the spectrum utilization efficiency [3].

As a generalization of OFDM, V-OFDM is a multi-carrier orthogonal modulation technology, and also has the disadvantage of high PAPR. So it is important and necessary

to research how to reduce PAPR in the V-OFDM system. In OFDM system, the study on PAPR reduction has been relatively mature. In general, the methods can be divided into three categories: distortion technology, coding technology, and non-distortion technology to reduce the PAPR of the OFDM system [4-9]. However, so far there is no the research on reduce PAPR in the V-OFDM is found. Coding technology has been applied in many systems for its perfect performance [10]. Reference [10] proposed a pre-coding technology to reduce PAPR in OFDM system, where the power of each modulated symbol is distributed over the OFDM block. A novel signal-to-leakage-and-noise-ratio (SLNR) pre-coding algorithm, which has the advantage of numerical stability and lower computational complexity, was proposed utilizing Fukunaga-Koontz (FK) transform in Multiple-Input-Multiple-Output (MIMO) systems [11]. Reference [12] proposed an encoding-pre distortion method in OFDM visible light communications (VLC-OFDM) system for PAPR reduction of VLC-OFDM system. However, the coding technology is only suitable for the situation where the number of subcarriers is less. Coding efficiency will decrease with the increment of number of subcarriers. The concept of subcarrier is substituted for VB in V-OFDM system [13], where fewer subcarriers are needed than OFDM system.

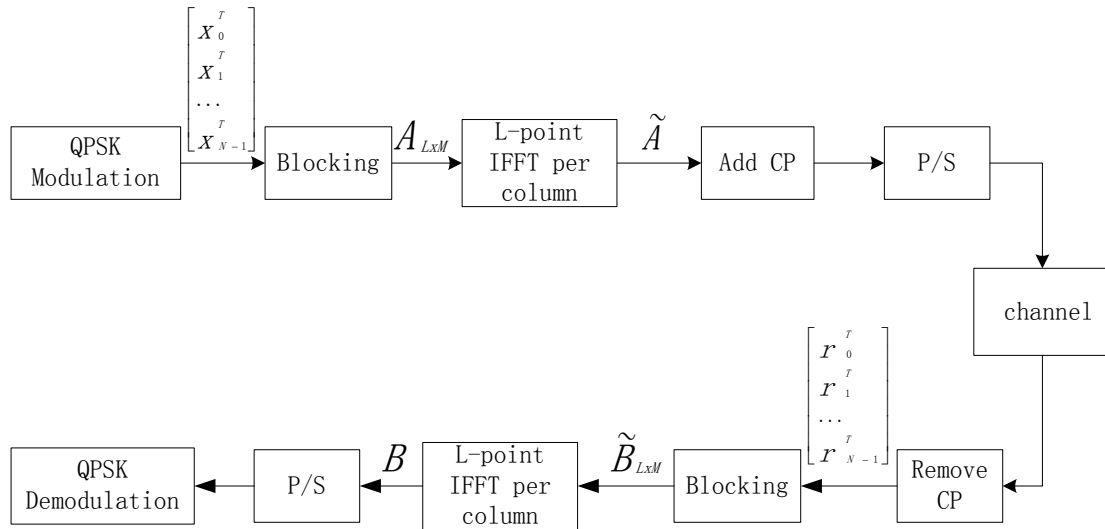
In this paper, we propose a modified pre-coding technology to reduce the PAPR in V-OFDM systems, where precoding consists of multiplying blocking (transform modulated data into a matrix) of each V-OFDM block by a precoding matrix before V-OFDM modulation (IFFT per

column) and transmission. Simulation results show that the modified pre-coding technology can reduce the PAPR of V-OFDM system more effectively.

This paper is organized as follows. In Section II, the V-OFDM system model is introduced. In Section III, we

describethe PAPRand modified pre-coding technology in V-OFDM system. The performance of modified pre-coding technology is analyzed in Section IV. In Section V, some conclusions are given.

**V-OFDM system model**



**Figure 1.**V-OFDM system model.

The V-OFDM system is a generalized version of the conventional OFDM system. The V-OFDM system model is depicted in Figure 1, where the modulated symbols are processed block by block. Different from conventional OFDM,V-OFDM further divides the length N block into L VBs, where each VB has size M .Assume that there are N = LM modulated symbols in one block, and denoted by  $x = [x_0, x_1, \dots, x_{N-1}]^T$ . At the transmitter, V-OFDM is to transform the traditional OFDMsymbols intoOFDM VBs, where the l-th row  $x_l$  is modulation symbol vector of the l-th VB in V-OFDM,  $x_l$  can be expressed as.

$$x_l = [x_{lM}, x_{lM+1}, \dots, x_{(l+1)M-1}]^T \quad (1)$$

The transmitter transforms the modulated symbols vector  $x = [x_0, x_1, \dots, x_{N-1}]^T$  into a  $L \times M$  matrix  $A$ , and performs L-point IFFT on per column of  $A$  and obtains a new matrix  $\tilde{A}$ . Then, the relationship between the m-th column of  $\tilde{A}$ , denoted by  $\tilde{x}_m$ , and  $x_l$  can be expressed as.

$$\begin{aligned} \tilde{x}_m &= \frac{1}{L} \sum_{l=0}^{L-1} x_l \exp\left(j \frac{2\pi ml}{L}\right), m = 0, 1, \dots, M - 1, l \\ &= 0, 1, \dots, L - 1 \quad (2) \end{aligned}$$

The last P column vectors of  $\tilde{A}$  are chosen as the cyclic prefix vectors. In order to avoid inter-block interference, the CP length P should larger than the maximum delay of the channel. After adding CP, the time domain signal is sent out serially by parallel-to-serial (P/S) transformation.

At the receiver, after removing CP, denote the received symbol vector by  $r = [r_0, r_1, \dots, r_{N-1}]^T$ , where the l-th row  $r_l$  is received symbol vector of the l-th VB in V-OFDM,  $r_l$  can be expressed as.

$$r_l = [r_{lM}, r_{lM+1}, \dots, r_{(l+1)M-1}]^T \quad (3)$$

The received symbol vector  $r = [r_0, r_1, \dots, r_{N-1}]^T$  is transformed into a  $L \times M$  matrix  $\tilde{B}$ , and take L-point FFT over per column of  $\tilde{B}$ . Then, the relationship between the

m-th column of  $\widetilde{r}_m$ , denoted by  $\widetilde{r}_m$ , and  $r_l$  can be expressed as.

$$\begin{aligned} \widetilde{r}_m &= \frac{1}{L} \sum_{l=0}^{L-1} r_l \exp\left(j \frac{2\pi ml}{L}\right), m = 0, 1, \dots, M-1, l \\ &= 0, 1, \dots, L-1 \end{aligned} \quad (4)$$

The relationship given by the reference [1] between the received symbol vectors  $r_l$  of V-OFDM and modulation symbol vector  $x_l$  can be expressed as.

$$r_l = H_l x_l + W_l \quad (5)$$

Where,  $H_l$  is a channel matrix of the l-th vector sub-channel  $W_l$  is the noise vector of the l-th vector sub-channel, whose entries are independent and identically distributed (i.i.d) and  $CN(0, \sigma^2)$ . No interference between each vector channel. The equivalent channel matrix  $H_l$  is given by.

$$H_l = (U_l)^H \overline{H}_l (U_l) \quad (6)$$

By the further deduce of the reference [4].

$$r_l = H_l x_l + W_l = (U_l)^H \overline{H}_l (U_l) x_l + (U_l)^H \overline{W}_l \quad (7)$$

Where,  $\overline{W}_l = [W_l, W_{l+L}, \dots, W_{l+(M-1)L}]^T$  is the additive white Gaussian noise vector,

$\overline{H}_l = \text{diag}(H_l, H_{l+L}, \dots, H_{l+(M-1)L})$  is  $M \times M$  diagonal matrix,  $U_l$  is an  $M \times M$  unitary matrix, whose entry in the s-th row ( $s=0, 1, \dots, M-1$ ) and m-th ( $m=0, 1, \dots, M-1$ ) column equals  $[U_l]_{s,m} = \frac{1}{\sqrt{M}} \exp\left\{-j \frac{2\pi}{N} m(l + sL)\right\}$ .

The formula (7) connects the V-OFDM system and traditional OFDM system.

### Modified Pre-Coding Technology to Reduce the PAPR of V-OFDM System

#### PAPR Problem of in V-OFDM Systems

V-OFDM can be regarded as a general vectorization of OFDM, for which the concept of subcarrier is substituted for VB. Different from OFDM, V-OFDM divides the length N block into L VBs, where each VB has size M. Assuming the

modulation symbol vector of V-OFDM system is  $[x_0, x_1, \dots, x_{N-1}]^T$ . Instead of doing N-point IFFT as in OFDM, V-OFDM does L-point IFFT over VBs. Assuming the signal before IFFT transform is a frequency domain signal  $x_l$ ,  $x(t)$  is a time domain signal obtained after IFFT transform in the V-OFDM system.  $x(t)$  can be written as.

$$x(t) = \frac{1}{\sqrt{L}} \sum_{l=0}^{L-1} x_l e^{j \frac{2\pi l t}{L}}, l = 0, 1, \dots, L-1 \quad (8)$$

Where, t represents the index number of sub-carrier, L is the number of VB.

PAPR of V-OFDM is the ratio of maximum peak power to the average power. It can be written as.

$$\text{PAPR(dB)} = \frac{P_{peak}}{P_{average}} = 10 \log_{10} \frac{\max_{0 \leq t \leq L-1} [|x(t)|]^2}{E[|x(t)|^2]} \quad (9)$$

Where,  $E[|x(t)|^2]$  is the mathematical expectation of V-OFDM signal power. PAPR of V-OFDM signal is a random variable.

Complimentary Cumulative Distribution Function (CCDF) is the most commonly used method as a measure of the PAPR reduction, and it represents the probability that the PAPR of VBs is larger than a certain threshold z. It is defined as.

$$\text{CCDF} = P(\text{PAPR} > z) = 1 - [1 - \exp(-z)]^L \quad (10)$$

Where z represents the threshold value of PAPR, L denotes the number of VB, the CCDF is a key performance indicator to measure a technology for PAPR reduction.

### Modified Pre-Coding Technology

Pre-coding technology is a processing method of reducing the PAPR of V-OFDM systems without distorting the signal. The data-sequence mapping changes the phase and amplitude distribution of original data, then the data is modulated and transmitted after pre-coding. Figure 2 illustrates the block diagram of the V-OFDM transmission system with precoder.

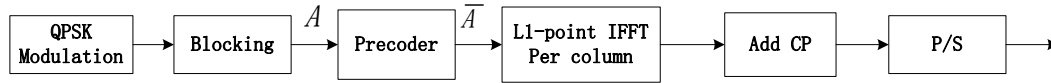


Figure 2. Transmitter block diagram of the modified pre-coding technology V-OFDM.

Pre-coding is a signal preprocessing technique. The pre-coder is applied between blocking and performing L1-point inverse fast Fourier transform (IFFT). The process can be considered as the frequency domain data transformation. References[10] proposed a pre-coding technique which had been used in OFDM system. Pre-coder is a  $N_1 \times N$  Precoding matrix, Where  $N$  is the number of subcarriers.  $N_1 = N + N_p$ ,  $N_1$  is the total number of subcarriers,  $N_p$  is the extension size, and  $N_p \in (0, N)$ . The pre-coding technique must be modified before being applied in V-OFDM systems because of existing differences between OFDM and V-OFDM systems. A modifying pre-coding technology based on the characteristics of V-OFDM system is proposed, and the explanation is as follows. Pre-coder is a  $L_1 \times L_p$  pre-coding matrix, where  $L_p$  is the number of VB, and  $L_1$  is the total number of VB.  $L_1 \times L$  Pre-coding matrix can be written as.

$$P = \begin{bmatrix} p_{0,0} & p_{0,1} & p_{0,2} & \dots & p_{0,L-1} \\ p_{1,0} & p_{1,1} & p_{1,2} & \dots & p_{1,L-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p_{L_1-1,0} & p_{L_1-1,1} & p_{L_1-1,2} & \dots & p_{L_1-1,L-1} \end{bmatrix} \quad (11)$$

Assume that  $L_1 = L + L_p$ , where  $L_1$  is the total number of VB,  $L_p$  is the extension size, and  $L_p \in (0, L)$ ,  $P_{a,b}$  is the first  $b$ -th VB, value of the first  $a$ -th sampling points separately.

In addition, the pre-coding matrix must satisfy  $P'P = I$ , where  $I$  is the  $L \times L$  identity matrix,  $P'$  is the conjugate transpose matrix of pre-coding matrix  $P$ .

According to the above pre-coding matrix (11), where the elements of the matrix  $P$  are defined as follows

$$P_{a,0} = \begin{cases} \frac{(-1)^a}{\sqrt{L}} \sin\left(\frac{a\pi}{2L_p}\right) & 0 \leq a \leq L_p \\ \frac{(-1)^a}{\sqrt{L}} & L_p \leq a \leq L \\ \frac{(-1)^a}{\sqrt{L}} \cos\left(\frac{(a-L)\pi}{2L_p}\right) & L \leq a \leq L_1 - 1 \end{cases} \quad (12)$$

Where  $L_1$  is the total number of VB. To evaluate the relationship between  $L$  and  $L_p$  in PAPR reduction, we define  $\beta$  as an expansion factor.

$$\beta = \frac{(L_1-L)}{L} = L_p/L \quad (13)$$

Then, the element  $P_{a,b}$  of pre-coding matrix can be expressed as.

$$P_{a,b} = P_{a,0} e^{-j2\pi \frac{ab}{L-1}} \quad (14)$$

Therefore, pre-coder and  $\bar{A}$  can be represented as

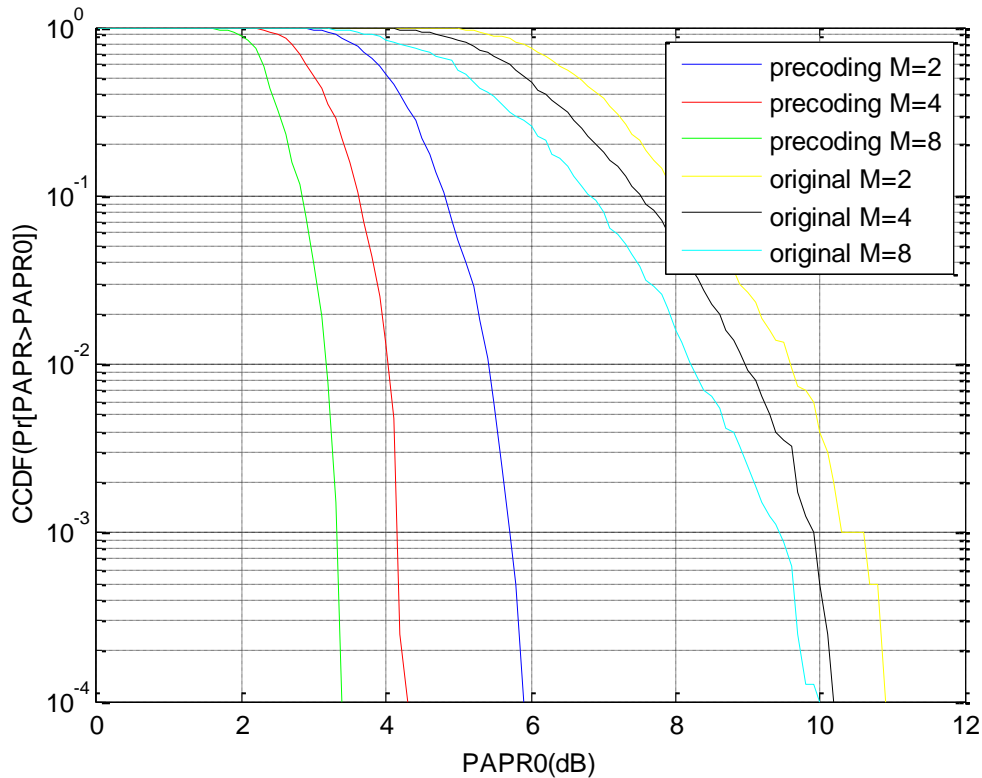
$$\bar{A} = PA = \begin{bmatrix} p_{0,0} & p_{0,1} & p_{0,2} & \dots & p_{0,M-1} \\ p_{1,0} & p_{1,1} & p_{1,2} & \dots & p_{1,M-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p_{L_1-1,0} & p_{L_1-1,1} & p_{L_1-1,2} & \dots & p_{L_1-1,M-1} \end{bmatrix} \quad (15)$$

The mathematical model shows that the modified pre-coding technology can be applied to the V-OFDM system to expand the original signal. Through the redistribution of data power from  $L$  to  $L_1$ , the phase distribution of each sampling point of the original V-OFDM symbol is changed to reduce PAPR.

### Simulation Results and Analysis

In this section, we perform simulations to validate the modified pre-coding technology, where the QPSK modulation scheme with gray mapping is employed. In the simulation, we assume that the V-OFDM symbol length is  $N=128$ , the CP length is  $P=12$ , and the channel with

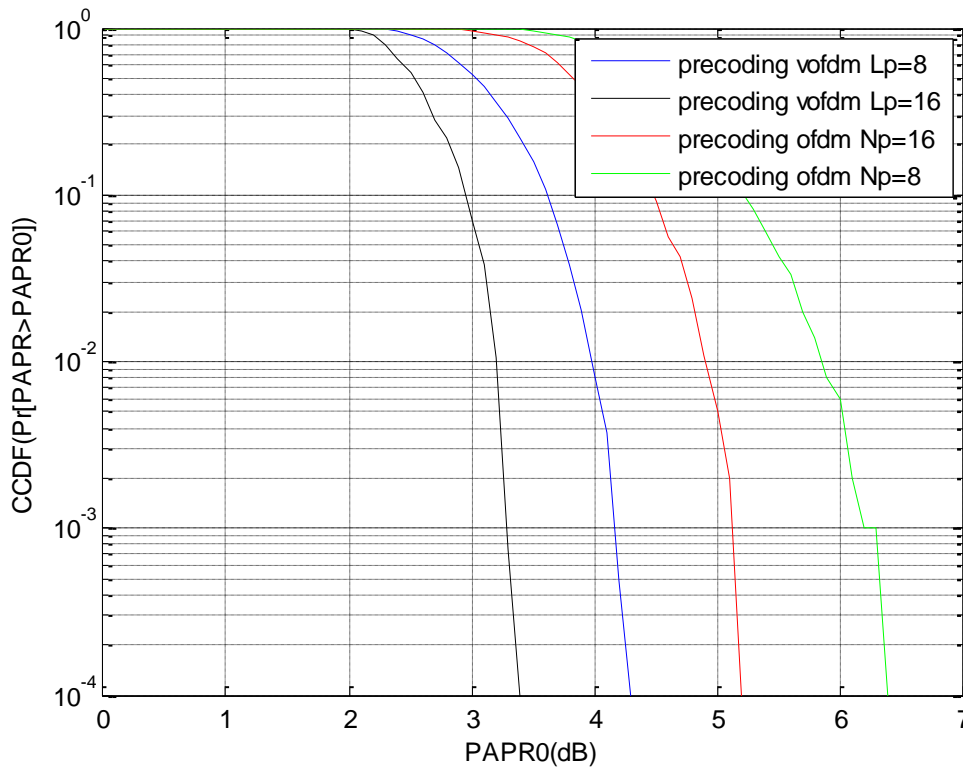
additive white Gaussian noise channel is employed. We use CCDF of PAPR to show the PAPR performance of V-OFDM system. Under the same condition of PAPR, smaller probability of PAPR shows better performance of PAPR reduction.



**Figure 3.** CCDF of the PAPR of modified pre-coding V-OFDM system and original V-OFDM system at different M.

Figure 3 compares CCDF of the PAPR of modified pre-coding V-OFDM system and original V-OFDM system at different M. The length of VBs, M, varies among 2, 4 and 8 with the IFFT length of L adjusted to keep  $N = LM$ . Compared with the PAPR of original V-OFDM, the PAPR of improved pre-coding V-OFDM can be reduced almost to 5.9 dB at  $CCDF = 10^{-4}$  with  $M = 2$  and  $\beta = 8/64$ , with 5 dB down. Compared with the PAPR of original V-OFDM, the PAPR of modified pre-coding V-OFDM can be reduced almost to 4.3 dB at  $CCDF = 10^{-4}$  with  $M = 4$

and  $\beta = 8/32$ , with 6 dB down. Compared with original V-OFDM, modified pre-coding V-OFDM can be reduced almost to 3.4 dB at  $CCDF = 10^{-4}$  with  $M = 8$  and  $\beta = 8/16$ , with 6.5 dB down. We can see from the Figure 3, While M increases from 2 to 8, L decreases from 64 to 16, the PAPR of the modified pre-coding V-OFDM system and original V-OFDM system are decreased rapidly. The PAPR of modified pre-coding V-OFDM system reduce with the increase of  $\beta$ .

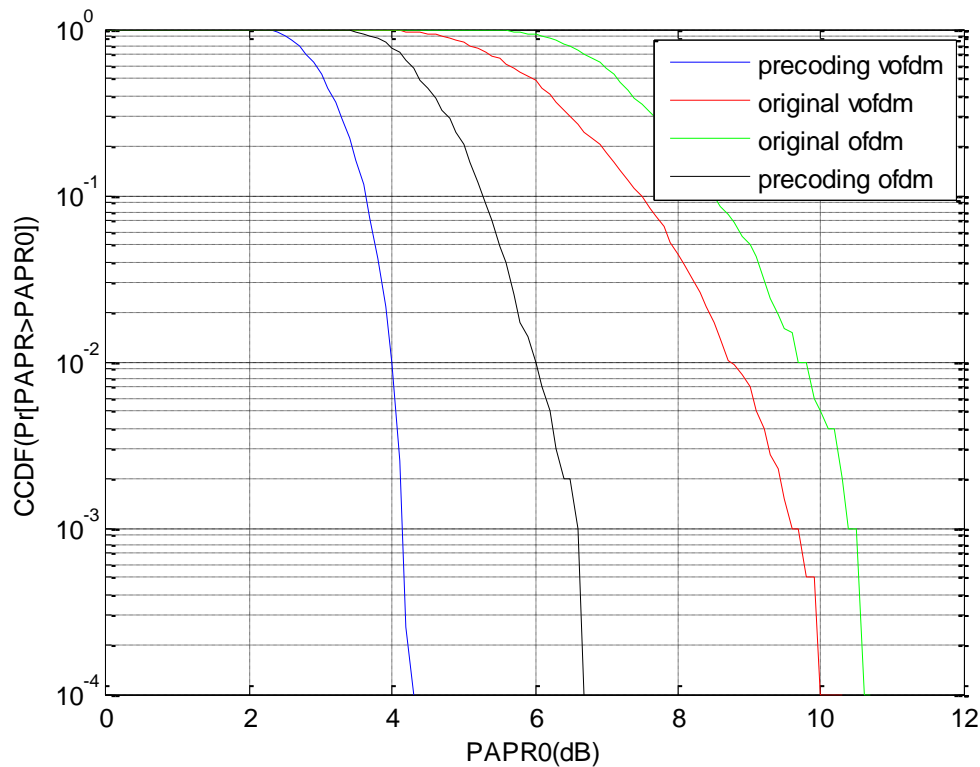


**Figure 4.**CCDF of the PAPR of modified pre-coding V-OFDM system and pre-coding OFDM system at different  $L_p$  and  $N_p$ .

Figure 4 compares CCDF of the PAPR of modified pre-coding V-OFDM system and pre-coding V-OFDM system at different  $L_p$  and  $N_p$ . A total of  $N=128$  subcarriers are considered in OFDM system.

The relationship given by the reference [10] between  $N_p$  and  $N$  can be expressed as  $\beta_1 = \frac{(N_1-N)}{N} = N_p/N$ . Compared with pre-coding OFDM system at  $N_p = 8$  with  $\beta_1=8/128$ , modified pre-coding V-OFDM system can be reduced almost to 4.3dB at  $CCDF = 10^{-4}$  with  $M = 4$

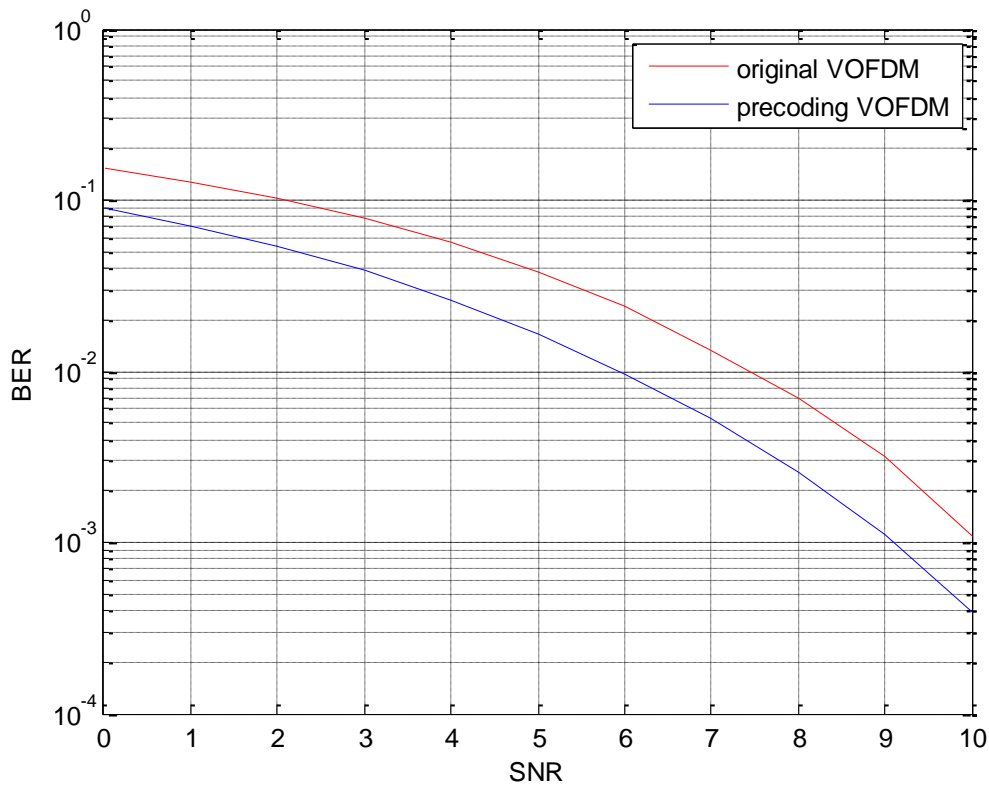
and  $L_p = 8$ ,  $\beta=8/32$ , with 2.1dB down. Compared with pre-coding OFDM system at  $N_p = 16$  with  $\beta_1=16/128$  modified pre-coding V-OFDM system can be reduced almost to 3.4dB at  $CCDF = 10^{-4}$  with  $M = 4$  and  $L_p = 8$ ,  $\beta=16/32$ , with 1.8dB down. We can see from the Figure 4, in the case of  $M = 4$ , the PAPR of modified pre-coding V-OFDM system would reduce with the increasing of  $L_p$ . The PAPR of pre-coding OFDM system would reduce with the increasing of  $N_p$ . Under the same condition of extension size, the modified pre-coding V-OFDM has a better PAPR performance than pre-coding OFDM systems.



**Figure 5.** CCDF of the PAPR Comparison between Modified pre-coding V-OFDM、original V-OFDM、 pre-coding OFDM and Original OFDM Systems.

Figure 5 compares CCDF of the PAPR of modified pre-coding V-OFDM、original V-OFDM、pre-coding OFDM and original OFDM system. Compared with original V-OFDM system, modified pre-coding V-OFDM system can be reduced almost to 4.3dB at  $CCDF = 10^{-4}$  with  $M = 4$  and  $\beta = 8/32$ , with 5.7dB down. Compared with original OFDM system, modified pre-coding V-OFDM system can be reduced almost to 4.3dB at  $CCDF = 10^{-4}$  with  $M = 4$  and  $\beta = 8/32$ , with 6.3dB down. Compared with pre-

coding OFDM system at  $\beta = 8/128$ , modified pre-coding V-OFDM system can be reduced almost to 4.3dB at  $CCDF = 10^{-4}$  with  $M = 4$  and  $\beta = 8/32$ , with 2.4dB down. We can see from the Figure 5, the modified pre-coding technology can be applied to a V-OFDM system directly, and reduce the PAPR of V-OFDM system at  $M = 4$  and  $\beta = 8/32$ . The modified pre-coding V-OFDM has a better PAPR performance than the original V-OFDM, pre-coding OFDM and original OFDM systems.



**Figure6.** BER comparison between Modified Pre-Coding V-OFDM and Original V-OFDM.

Figure 6 compares the BER of modified pre-coding V-OFDM system and original V-OFDM system. Over an additive white Gaussian noise (AWGN) channel, comparing with the original V-OFDM system, the modified pre-coding V-OFDM system has a 1 dB gain on signal noise ratio (SNR) when the bit error rate (BER) reached  $10^{-3}$  in the case of  $M = 4$ . We can see from the Figure 6, the BER performance of the modified pre-coding V-OFDM system is better than the original V-OFDM system.

### Conclusion

We propose a modified pre-coding technique to reduce the PAPR of V-OFDM system. The proposed technique is based on signal pre-coding, where blocking of each V-OFDM block is multiplied by a precoding matrix before V-OFDM modulation and transmission. This technology is data-independent and, thus, does not require new processing and optimization for each transmitted V-OFDM block. The

Simulation results show that the PAPR of V-OFDM system reduces with the increase of  $M$ . In the case of  $M = 4$ , the modified pre-coding V-OFDM has better PAPR performance than the pre-coding OFDM and original OFDM systems. The modified pre-coding V-OFDM system has a 1 dB gain on signal noise ratio (SNR) when the bit error rate (BER) reached  $10^{-3}$  in the case of  $M = 4$ . In a word, the modified pre-coding technology is a good solution to the PAPR reduction of V-OFDM system.

### Acknowledgment

This work was financially supported by Natural Science Foundation of China (Grant NO. 61172047 and 91123035), Xinjiang Provincial Natural Science Foundation (Grant NO. 2013211A035), Open Foundation from Metal Resources Strategic Research Institute of Central South University, and Hunan Provincial Natural Science Foundation (Grant NO. 14jj2013).



## References

1. Xia X G. Precoded and vector OFDM robust to channel spectral nulls and with reduced cyclic prefix length in single transmit antenna systems. *IEEE Transactions on Communications* 2001; 49(8); 1363-1374.
2. Hong Zhang, Xiang-Gen Xia. A guard band configuration scheme for single-antenna vector-OFDM systems. *IEEE Conference Record of the Thirty-Eighth Asilomar Conference on Signals, Systems and Computers* 2004; 1; 827-831.
3. Hong Zhang, Xiang-Gen Xia. Iterative decoding and demodulation for single-antenna vector-OFDM systems. *IEEE Transactions on Vehicular Technology* 2006; 55(4); 1447-1454.
4. SeungHee Han, Jae Hong Lee. "An Overview of Peak-To-Average Power Ratio Reduction Techniques for Multi-carrier Transmission," *IEEE wireless Communications*. 2005; 12 (2); 56-66.
5. K. Bandara, P. Niroopan, Chung Yeon-Ho, "PAPR reduced OFDM visible Light communication using exponential nonlinear companding" in *Proc. IEEE Int. Conf. Microw. Commun. Antennas Electron. Syst. (COMCAS)* 2013; 1-5.
6. Tao Jiang, Yang Yang, Yong-Hua Song, Exponential Companding Technique for PAPR Reduction in OFDM Systems, *IEEE Transactions on Broadcasting* 2005; 51(2); 244-248.
7. Inoue, Y.; Tsutsui, H.; Miyanaga, Y. Study of PAPR reduction using coded PTS in 8×8 MIMO-OFDM systems. *Intelligent Signal Processing and Communications Systems* 2013; 363-368.
8. L. Wang, C. Tellambura, "A simplified clipping and filtering technique for PAPR reduction in OFDM Systems," *IEEE Signal Process. Lett* 2005; 12(6); 453-456.
9. Cimini, L. J., Sollenberger, N. R., "Peak-to-average power ratio reduction of an OFDM signal using partial transmit sequences," *IEEE Communications Letters* 2000; 4(3); 86-88.
10. Slimane Ben Slimane. Reducing the peak-to-average power ratio of OFDM signals through precoding. *IEEE Transactions on vehicular technology* 2007; 56(2); 686-695.
11. Zuoliang Yin, Xing peng Mao, Nai tong Zhang. An FK-transform based MU-MIMO precoding with low complexity. *Journal of Harbin Engineering University* 2013; 34(3); 375-380.
12. Hong gui Deng, Yi Sun, Chen Yang. "A New Method to Reduce Peak to Average Power Ratio in Optical OFDM Communication," *Science Journal of Physics* 2013; 1-6.
13. Peng Cheng, Meixia Tao. V-OFDM: on performance limits over multi-path Rayleigh fading channels. *IEEE Trans on Communications* 2011; 59(7); 1878-1892.

## List Of Figure Captions

Figure 1. V-OFDM system model.

Figure 2. Transmitter block diagram of the modified pre-coding technology V-OFDM.

Figure 3. CCDF of the PAPR of modified pre-coding V-OFDM system and original V-OFDM system at different M.

Figure 4. CCDF of the PAPR of modified pre-coding V-OFDM system and pre-coding OFDM system at different  $L_p$  and  $N_p$ .

Figure 5. CCDF of the PAPR comparison between modified pre-coding V-OFDM, original V-OFDM, pre-coding OFDM and original OFDM systems.

Figure 6. BER comparison between modified pre-coding V-OFDM and original V-OFDM.