The Amelioration of Einstein’s Equation and New Steady Cosmology

Yang Ming *, Yang Jian Liang
Department of Physics, Zhengzhou University, China
* corresponding author: a2937061@163.com, “ming yang” <bps26@sina.cn>

Abstract:
In the framework of gravitational theory of general relativity, this article has systematically and radically solved the problem of galaxy formation and some significant cosmological puzzles. A flaw with Einstein’s equation of gravitational field is firstly corrected and the foundations of general relativity are perfected and developed, and space-time is proved to be infinite, expansion and contraction of universe are proved to be in circles, the singular point of big bang is naturally eliminated, celestial bodies and galaxies are proved growing up with cosmic expansion, for example Earth’s mass and radius at present increase by 1.2 trillion tons and 0.45mm respectively in a year, in response to which geostationary satellites rise by 2.7mm.

PACS: 04.20.Jb, 04.25.-g, 98.80.Jk, 95.30.Sf.

Key words: Background Coordinates; Standard coordinates; Geodesic; negative pressure

I. Introduction.

Though general relativity obtains considerable success, some significant fundamental problems such as the problem of singular point, the problem of horizon, the problem of distribution and existence of dark matter and dark energy, the problem of the formation of celestial bodies and galaxies, the mystery of solar neutrino, as well as the problem of asymmetry of particle and antiparticle, always are not solved naturally and satisfactorily. These problems long remain implies strongly that the fundamentals of general relativity have flaw and needs further perfection. For the purpose, this paper begins with determining the vacuum solution of Einstein’s field equation in the background coordinate system, then by correcting rationally Einstein’s field equation from an all new perspective these get problems removed.

II. The static metric of spherical symmetry in background coordinate system.

In this paper light’s speed  $c = 1$. According to general relativity, for the static and spherically symmetric case, in the standard coordinate system (Weinberg, S. 1972; Peng, 1998), the correct form of invariant line element outside gravitational source is given by

$$ds^2 = -d\tau^2 = \left(1 - \frac{2GM}{l}\right)dt^2 + \left(1 - \frac{2GM}{l}\right)^{-1}dl^2 + l^2(d\theta^2 + \sin^2 \theta d\phi^2)$$  \hspace{1cm} (1)

which is called Schwarschild metric and satisfies vacuum field equation $R_{\mu\nu} = 0$ with $t, l, \theta, \phi$ as independent coordinates. Here $\tau$ is proper time, $M$ is the total mass of gravitational source; $l$ is usually explained as standard radial coordinate, which doesn’t have clear physical meaning and only in the far field is approximately viewed as true radius. In order to describe clearly dynamic behavior and definite position of a particle in gravitational field and enable general relativity to have common language with other theories including Newton’s gravitational theory and compare results with one another, it is necessary to transform line element (1) into the form expressed in Science
background coordinates. Hence we take \( l = l(r) \). In this paper \( r \) is defined as background coordinate (Zhou, 1983; Fock, 1964) and refers to true radius, that is to say, its meaning is the same as that used usually in quantum mechanics or electrodynamics. \( t, \theta, \phi \) are standard coordinates and can also be viewed as background coordinates, which represent true time and angle. In the following we try to determine \( l = l(r) \) by the introducing an additional transformation equation, and such operation is allowed is because metric tensor satisfies Bianchi identity and if a metric is a solution of field equation in one coordinate system it is also a solution under arbitrary coordinate transformation, and the meaning of applying coordinate transformation is to guarantee the new metrics meet field equation.

According to general relativity the dynamical equation of particle outside source is geodesic equation

\[
\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0
\]

Note that repeated indexes up and down mean sum. For the convenience of practical application, especially relate to solving acceleration of moving particle, the proper time \( \tau \) need be eliminated, and it is easy

On one hand

\[
\frac{d^2x^\mu}{d\tau^2} = \frac{dt}{d\tau} \frac{d}{dt} \left( \frac{dt}{d\tau} \frac{dx^\mu}{dt} \right) = \left( \frac{dt}{d\tau} \right)^2 \frac{d^2x^\mu}{dt^2} + \frac{d^2t}{d\tau^2} \frac{dx^\mu}{dt}
\]

On the other hand

\[
\Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = \left( \frac{dt}{d\tau} \right)^2 \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{dt} \frac{dx^\beta}{dt}
\]

and adding them and using the above geodesic equation give immediately the following equivalent geodesic equation

\[
\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} - \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{dt} \frac{dx^\beta}{dt} = 0
\]

where \( x^0 = t \), and note that in this paper indexes \( \lambda, \nu, \mu, \sigma, \alpha, \beta = 0, 1, 2, 3 \). Eq. (2) can exist in any coordinate system and is a basic equation of general relativity, which free particles in gravitational field must satisfy.

When a particle of mass \( m \) is moving along radius in the static gravitational field of spherical symmetry, giving consideration to the effect of its speed, in the background coordinate system, in the far field (weak field) the radial component of Eq. (2) should reduce to the following relativistic dynamic Eq. (3) rather than others

\[
\frac{d}{dt} \left( \frac{dr}{dt} \right) m = -\frac{mGM}{r^2}
\]

where \( m \) refers to relativistic dynamic mass, namely \( m = \frac{m_0}{\sqrt{1 - v^2}} \). Why the radial component should reduce to (3) is that (3) stands for the equality of gravitational mass and inertial mass and also stands for the speed of light is the limit one. In order to enable it to reduce to (3) we may introduce a transformation equation as follows

\[
\frac{dl}{dr} = \sqrt{1 - \frac{2GM}{l}} \exp \left( -\frac{GM}{r} \right)
\]

The correctness of Eq. (4) will be seen later, it determines a coordinate transformation of \( l \to r \). By means of separating variables, the solution of Eq. (4) is easily given by

\[
\sqrt{l(l-2GM)} + 2GM \ln \left( \sqrt{l} + \sqrt{l-2GM} \right) = C_i + r - GM \ln r - \frac{1}{2r} G^2 M^2 + \frac{1}{12r^2} G^3 M^3 + \cdots
\]
Here constant $C_1$ is determined according to the continuity of function $l = l(r)$ on the boundary of source, and the back Eq. (23) can give out the boundary value $l(r_0)$, $r_0$ denotes source’s radius (celestial body radius). Note that (5) makes sure $l \approx r$ for $r \to \infty$, prove as follows.

Form Eq. (4) we see $l \to \infty$ for $r \to \infty$, and considering of $\lim_{x \to \infty} \frac{\ln x}{x} = 0$, it holds that for $l \to \infty$ the left-hand side of (5) is

$$l \left( \sqrt{1 - \frac{2GM}{l}} + \frac{GM}{l} \ln l + \frac{2GM}{l} \ln \left( 1 + \sqrt{1 - \frac{2GM}{l}} \right) \right) \approx l,$$

and for $r \to \infty$, the right-hand side of (5) is

$$r \left( \frac{C_1}{r} + 1 - \frac{GM}{r} \ln r - \frac{G^2M^2}{2r^2} + \frac{G^3M^3}{12r^3} + \cdots \right) \approx r.$$

Under transformation of Eq. (4), the (1) becomes the following (5) which is an exact solution of vacuum field equation $R_{\mu\nu} = 0$ in the background coordinate system $x^\mu = (x^0, x^1, x^2, x^3) = (t, r, \theta, \phi)$.

$$ds^2 = -\left( 1 - \frac{2GM}{l} \right)dt^2 + \exp(-\frac{2GM}{r})dr^2 + l^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right). \quad (6)$$

Note that now $l = l(r)$ is already a concrete function of $r$, which is decided by (5) and can not be written out explicitly. And here $t, r, \theta, \phi$ are independent coordinate variables.

In the far field, the line element (6) provides $g_{00} = -1 + \frac{2GM}{l(r)} \approx -1 + \frac{2GM}{r}$, $g_{11} = \exp(-\frac{2GM}{r}) \approx 1 - \frac{2GM}{r}$,

$g_{22} = l^2(r) \approx r^2$, $g_{33} = l^2(r) \sin^2 \theta \approx r^2 \sin^2 \theta$, $\Gamma^0_{01} \approx \frac{GM}{r^2}$, $\Gamma^1_{01} \approx \frac{GM}{r^2}$, $\Gamma^0_{11} \approx \frac{GM}{r^2}$, $\Gamma^0_{00} = 0$, $\Gamma^0_{11} = 0$, $\Gamma^0_{00} = 0$,

and introducing them into (2) and putting $\mu = 1$, $d\theta = d\phi = 0$, $v = \frac{dr}{dt}$, we obtain

$$\frac{d^2 r}{dt^2} + \left( 1 - v^2 \right) \frac{GM}{r^2} = 0,$$

which is equivalent to Eq. (3). Proof: assume $d\theta = d\phi = 0$, $m = \frac{m_0}{\sqrt{1 - v^2}}$, from equation (3) we have

$$0 = \frac{d}{dt} \left[ \left( \frac{dr}{dt} \right)^2 \right] + \frac{mGM}{r^2} = m_0 \left[ \frac{d}{dt} \left( 1 - v^2 \right)^{-1/2} \frac{dv}{dt} + (1 - v^2)^{-1/2} \frac{dv}{dt} \right] + \frac{mGM}{r^2}$$

$$=(1 - v^2)^{-3/2} \frac{d^2 r}{dt^2} m_0 + m \frac{GM}{r^2} = m \left[ (1 - v^2)^{-1} \frac{d^2 r}{dt^2} + \frac{GM}{r^2} \right],$$

which immediately yields (7).

By far, we may say that (6) is just the appropriate line element expressed in background coordinate system $x^\mu = (x^0, x^1, x^2, x^3) = (t, r, \theta, \phi)$, which we look for and satisfies vacuum field equation and entire requirements on physics.

Obviously it is, however neglected usually, necessary to identify which of the solutions that satisfy field equation in the same coordinate system is correct or more correct. As a example worthy of mentioning, we point out that applying directly $l = r$ in (1), namely $l$ is directly explained as background coordinate, gives the following.
Another exact solution expressed in the same background coordinate system $x^\mu = (x^0, x^1, x^2, x^3) = (t, r, \theta, \varphi)$

$$ds^2 = -\left(1 - \frac{2GM}{r}\right)dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1}dr^2 + r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2). \quad (8)$$

However, in accordance with (8) the corresponding geodesic can't reduce to (3) in weak field, instead it reduces to

$$\frac{d^2r}{dt^2} + \frac{(1 - 3v^2)GM}{r^3} = 0 \quad (9)$$

Proof: (8) provides $g_{00} = -1 + \frac{2GM}{r}$, $g_{11} = \left(1 - \frac{2GM}{r}\right)^{-1}$, $g_{22} = r^2$, $g_{33} = r^2 \sin^2\theta$, $g_{\mu\nu} = 0(\mu \neq \nu)$,

$$\Gamma^i_{01} = \frac{1}{2}g^{i\rho}(\partial g_{0\rho})/\partial x^i + \partial g_{0i}/\partial x^\rho - \partial g_{i\rho}/\partial x^0 = -\frac{GM}{(1 - 2GM/r)r^2}$$. \quad $\Gamma^0_{01} = \frac{GM}{(1 - 2GM/r)r^2}$, \quad $\Gamma^i_{00} = \frac{(1 - 2GM/r)GM}{r^2}$

$\Gamma^i_{01} = 0$, substituting them into (2) and taking $\mu = 1$ and $d\varphi = d\theta = 0$ yield immediately

$$\frac{d^2r}{dt^2} = -\Gamma^i_{00} - \Gamma^i_{11} v^2 + 2v^2\Gamma^0_{01} = -(1 - \frac{2GM}{r})\frac{GM}{r^2} + \frac{3GM}{(1 - 2GM/r)r^2}v^2$$, and for $\frac{2GM}{r} \ll 1$, this equation distinctly reduces to Eq. (9), which isn't Eq. (3). It is easily found that Eq. (9) not only goes against the elementary principle of equality of gravitational mass and inertial mass but also leads to incorrect conclusion that gravitational field becomes repulsive one for a particle whose speed exceeds 0.58c. Hence Eq. (9) must be wrong, and implies (8) can't describe high speed and has a certain shortcoming compared with (6).

From the wrong Eq. (9) that line element (8) implies we understand why $l$ in (1) cannot endow the meaning of background coordinate.

Note that the angle of orbital precession of Mercury described by (6) is still the same as that described by line element (8) (Peng, 1998), the angle of orbital precession doesn't change under the transformation of radial coordinates. On all accounts, (6) is the correct line element expressed in background coordinate system.

And again, though general relativity is fully covariant and can use all sorts of coordinates, we must use background coordinates when we take geodesic equation to compare with Newtonian gravitational law which is expressed in background coordinates, otherwise they don't have the common language and the meaning of each term in geodesic equation is unclear and the comparison is distinctly ruled out. This shows that the special advantage of using background coordinates that have clear physical meaning. And certainly, using background coordinates general relativity becomes naturally flat space-time’s gravitational theory and combines practice more intuitively and has common language with other theory of physics. In a word, using background coordinates the coordinate’s derivatives with respect to time represent speed and acceleration we can directly decide acceleration of a particle by solving Eq. (2).

In terms of the observational theory of general relativity, so-called background coordinates are just the values measured by the rest observer in the distance, and as for $r$, it is just the length from origin of coordinates to another point, which is measured by the observer, and $t, \theta, \varphi$ are the time and angle respectively, which are measured by the observer.

Of course, on earth using which sort of coordinates is in accordance with specific conditions and questions to demand to solve, and sometimes we have to use the sort of coordinates whose physical meaning is not too clear in order to simplify mathematical calculation, but this certainly misses out or covers up some important information and even can not link theory with observations.
Finally must point out: though Schwarzschild standard radial coordinate isn’t explained as background coordinate (namely true radius) in standard textbooks one treats it as true radius involuntarily in practice, this makes certain confusion on logic and concept. For example, while computing deflected angle of light on Sun’s surface, one takes the value of Schwarzschild standard radial coordinate on the surface for Sun’s true radius, serious question doesn’t happen thanks to the difference between $l$ and $r$ slight (see the calculated result in section V). In this paper, in order to hint the difference on concept Schwarzschild radial coordinate is denoted by $l$ and true radius is denoted by $r$, therefore this paper is actually to perfect and refining the fundamentals of general relativity. As a result of careful calculation step by step, we find Einstein’s field equation may change, and by applying the revised field equation we see that many difficult problems of cosmology can all be readily solved and maybe new physics will be brought out.

### III. The Amelioration of Einstein’s gravitational field equation.

It is seen from the above discussions that in spherically symmetric gravitational field, in the case of weak field approximation, $g_{\infty} = -1 + \frac{2GM}{r}$ and $g_{11} = 1 - \frac{2GM}{r}$ instead of the previous $g_{11} = 1 + \frac{2GM}{r}$, which guarantee Eq. (7) can appear and hint us to alter the coupling constant $\gamma$ in field equation $R_{\mu\nu} = \gamma(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu})$. This is because the coupling constant $\gamma$ relates to the form of weak field approximation metrics $g_{\mu\nu}$ and is confirmed in the course of solving weak field approximation metrics, and the change of the metrics means the coupling constant $\gamma$ need also change. So, the content of the section III is actually to renew solving under certain condition Einstein’s field equation and in the same course decide the coupling constant $\gamma$.

And now we set out to reconfirm the coefficient $\gamma$ by solving weak field approximation metrics $g_{\mu\nu}$. Here energy-momentum tensor $T_{\mu\nu} \equiv (\rho + p)U_\mu U_\nu + pg_{\mu\nu}$, and four contravariant speed $U_\mu \equiv \frac{dx^\mu}{d\tau}$, corresponding covariant speed $U_\mu \equiv g_{\mu\nu} U^\nu$. And from $ds^2 = -d\tau^2 = g_{\mu\nu} dx^\mu dx^\nu$, we have $U_\mu U^\mu = -1$, hence it follows that

$$T \equiv g^{\mu\nu} T_{\mu\nu} = g^{\mu\nu} (\rho + p)U_\mu U_\nu + pg^{\mu\nu} g_{\mu\nu} = (\rho + p)U_\mu U^\mu + 4p = 3p - \rho$$

Here pressure $p$ isn’t assumed as zero in advance and it is also to be solved. Similar to previous calculation used in standard textbooks, the following discussions are still carried out in the background right-angled coordinate system $x^\mu = (x^0, x^1, x^2, x^3) = (t, x, y, z)$. And in the coordinate system, for weak field we have $g_{\mu\nu} = h_{\mu\nu} + \eta_{\mu\nu}$ and $|h_{\mu\nu}| << 1$ (note that only in such coordinate system these can exist). Here Minkowskian metrics $\eta_{00} = -1$, $\eta_{11} = \eta_{22} = \eta_{33} = 1$, and the other $\eta_{\mu\nu} = 0$ $(\mu \neq \nu)$. Omitting these terms of less than $o(h^2)$ we have the following relations (Weinberg, S. 19720)

$$\Gamma^\nu_{\alpha\beta} = \frac{1}{2} \eta^{\alpha\nu}(\frac{\partial g_{\rho\mu}}{\partial x^\beta} + \frac{\partial g_{\rho\beta}}{\partial x^\mu} - \frac{\partial g_{\mu\beta}}{\partial x^\rho})$$

$$h_{\mu\nu} = \eta^{\mu\nu} h_{\rho\sigma}, \text{ and } h = h_{\mu\nu} = \eta^{\mu\nu} h_{\rho\sigma}.$$ Correspondingly, Rich tensor

$$R_{\mu\nu} = \Gamma^\sigma_{\mu\sigma\nu} - \Gamma^\sigma_{\mu\nu\sigma} = \frac{1}{2} \eta^{\sigma\Sigma} h_{\mu\nu,\lambda,\sigma} + \frac{1}{2} (h_{\mu,\nu} - h_{\mu,\nu} - h_{\nu,\mu} - h_{\sigma,\gamma} - h_{\gamma,\sigma} - h_{\sigma,\gamma})$$

where the semicolons denote covariant derivative and the commas denote common derivative.

May as well use harmonic condition (as we used to do in standard textbook)

$$h_{\mu,\nu} = \frac{1}{2} h_{\mu,\nu}$$

Differentiating Eq. (10) with respect to $x^\nu$ yields

$$h_{\mu,\nu} = \frac{1}{2} h_{\mu,\nu}.$$ Similarly, $h_{\mu,\nu} = \frac{1}{2} h_{\nu,\mu}$. Using

$$h_{\mu,\nu} = \frac{1}{2} h_{\mu,\nu} \text{ and adding up the above two equations yield } h_{\mu,\nu} - h_{\mu,\nu} - h_{\nu,\mu} - h_{\sigma,\gamma} - h_{\gamma,\sigma} - h_{\sigma,\gamma} = 0.$$
Hence, we obtain $\nabla^2 h_{\mu\nu} - \frac{\partial^2 h_{\mu\nu}}{\partial t^2} = 2\gamma(T_{\mu\nu} - \frac{1}{2} T\eta_{\mu\nu}) = 2\gamma[(\rho + p)U_\mu U_\nu + \frac{\rho - p}{2} \eta_{\mu\nu}]$, which have retarded solutions $h_{\mu\nu} = -\frac{\gamma}{4\pi} \int \frac{2(\rho + p)U_\mu^2 + (\rho - p)\eta_{\mu\nu}}{\xi} \, dx' dy' dz'$. Here $\xi = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$, $i, j, k = 1, 2, 3$, the terms in the integral sign take the values of $t' = t - \xi$. Note that the above retarded solutions can be used in arbitrary cases of motion of source. Hence, in order to get the external metrics $g_{00} = -1 + \frac{2GM}{r}$ and $g_{ii} = -1 - \frac{2GM}{r}$ in the case of static spherical symmetry ($U_\mu = \eta_{0\mu} U^\mu = -1$, $U_j = 0$), which make sure that the geodesic equation of a moving particle along coordinate axis or radial direction can reduce to Eq. (3), it must be required that the constant coefficient $\gamma$ is equal to $4\pi G$ and simultaneously pressure $p$ satisfies

$$\int \frac{p}{\xi} \, dx' dy' dz' = -\frac{M}{r} \quad \text{for} \quad r = \sqrt{x^2 + y^2 + z^2} \geq r_0,$$

which means

$$\int p \, dx dy dz = -\int \rho \, dx dy dz = -M. \quad (11)$$

In view of (3) it must hold that $h_{0j} = 0$ in the static case. Next we solve the other three $h_{ij}$. Inserting

$$h_{\mu\nu} = \eta^{\alpha\beta} h_{\alpha\beta} \quad \text{and} \quad h = \eta^{ij} h_{ij} = -h_{00} + 3h_1 \quad \text{into (10)},$$

and noticing $h_{11} = h_{22} = h_{33}$, $h_{ij} = h_{ji}$, $h_{ij}^0 = h_{ij}^0 = 0$, we obtain three equations as follows

$$h_{111} + h_{222} = \frac{1}{2}(h_{11} - h_{00}),$$

$$h_{121} + h_{232} = \frac{1}{2}(h_{11} - h_{00}),$$

$$h_{122} + h_{233} = \frac{1}{2}(h_{11} - h_{00}).$$

After a certain calculation we arrive

$$h_{ij} = \frac{1}{4} \left[ (h_{11} - h_{00})_i j + (h_{11} - h_{00})_j i - (h_{11} - h_{00})_k, j_k \right].$$

Here $i \neq j$, $i \neq k$, $k \neq j$, and $i, j, k = 1, 2, 3$. With the condition $h_{ij} \to 0$ for $r \to \infty$, $h_{ij}$ are solved by

$$h_{ij} = \frac{1}{4} \int_{x^1}^{x^1} \int_{x^i}^{x^i} \left[ \frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial (x^i')^2} - \frac{\partial^2}{\partial (x^j')^2} (h_{11} - h_{00}) \right] \, dx^i \, dx^i.$$

Note that $x^1 = x, x^2 = y, x^3 = z$. On the other hand, for the weak field case Bianchi identity can give the ordinary
conservation law \( T^{\mu}_{\nu,\mu} = 0 \).

Proof: because \( R_{\nu,\mu}^{\mu} = R_{\nu,\mu}^{\mu} + \Gamma^{\mu}_{\delta\mu}R_{\nu,\delta}^{\delta} - \Gamma^{\mu}_{\delta\nu}R_{\nu,\mu}^{\delta} = R_{\nu,\mu}^{\mu} + o(h^2) = R_{\nu,\mu}^{\mu} \), then

\[
0 = (R_{\nu,\mu}^{\mu} - \frac{1}{2}R\delta^{\mu})_{,\mu} = R_{\nu,\mu}^{\mu} - \frac{1}{2}R_{,\nu} = R_{\nu,\mu}^{\mu} - \frac{1}{2}R_{,\nu},
\]
and moreover field equation gives \( R = -\gamma T \).

\[
R_{\nu,\mu} = \gamma(T_{\nu,\mu}^{\mu} - \frac{1}{2}T\delta^{\mu})_{,\mu} = \gamma(T_{\nu,\mu}^{\mu} - \frac{1}{2}T_{,\nu}) = \gamma T_{\nu,\mu}^{\mu} + \frac{1}{2}R_{,\nu},
\]
hence \( T_{\nu,\mu}^{\mu} = 0 \).

And for the static case, using \( T_{\nu,\mu}^{\mu} = [(\rho + p)U_{\nu}U^{\mu}]_{,\mu} + (p\delta^{\mu})_{,\mu} = 0 \) yields \( \frac{\partial p}{\partial x^\varepsilon} = 0 \), considering of

\[
\nabla^2(h_{00} - h_{ii}) = 16\pi Gp,
\]

it is immediately verified that

\[
\nabla^2h_{ij} = \frac{1}{4} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ \left( \frac{\partial^2}{\partial x^j} + \frac{\partial^2}{\partial x^i} - \frac{\partial^2}{\partial (x^j)^2} \right) \nabla^2(h_{11} - h_{00}) \right] dx^i dx^j = 0
\]

That is to say, \( h_{ij} \) worked out here is indeed reasonable approximate solution of field equation with \( \gamma = 4\pi G \).

And again, as a special case of spherical symmetry, if the source's density is a constant, namely \( \frac{\partial \rho}{\partial x^\varepsilon} = 0 \), since \( \frac{\partial p}{\partial x^\varepsilon} = 0 \) we can infer from (11) a very useful and significant result

\[
p = -\rho
\]

which can be regarded as the form of pressure in weak field in the case of that density \( \rho \) is even. It is obviously too subjective to take gravitational source's pressure for zero in advance, in fact, by intense calculation we see that the pressure takes negative value where matter exists and the places where matter exists turn out to be so-called pseudo-vacuum (Gondolo, P. 2003; Guth, 1981). And obviously the pressure as gravitational source isn't so-called thermodynamic pressure.

To sum up, we can conclude that in any coordinate system gravitational field equation is revised as

\[
R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 4\pi GT_{\mu\nu},
\]

where positive 4 replaces the previous \( -8 \), obviously Eq. (12) preserves general covariance.

Of course, line element (6) satisfies Eq. (12) because both \( p \) and \( \rho \) vanish outside gravitational source and Eq. (12) becomes the vacuum field \( R_{\mu\nu} = 0 \) outside source, whose form is the same as the previous.

IV. Applications and tests of Eq. (12) in cosmology.

It is decided by practice in the final analysis whether a theory is right or not. The application of Eq. (12) in cosmology proves strongly that the revision is successful.

With \( l \) as standard radial coordinate, in the co-moving coordinates Friedmann-Robertson-Walker metric is given by (Weinberg, S. 1972; Sawangwit, U.2005)

\[
ds^2 = -dt^2 + a^2(t) \left[ \frac{1}{1-kl^2}dl^2 + l^2d\theta^2 + l^2\sin^2\theta d\phi^2 \right]
\]
Here $a(t)$ is universe expansion factor, and metric $g_{00} = -1$, $g_{11} = \frac{a^2(t)}{1-kl^2}$, $g_{22} = a^2(t)l^2$,
\[ g_{33} = a^2(t)l^2 \sin^2 \theta, \quad g_{\mu\nu} = 0 (\mu \neq \nu), \]
and substituting they into (11) yields the following equation like Friedmann’s
\[ \left( \frac{da(t)}{dt} \right)^2 + k = -\frac{4\pi G}{3} \rho a^2(t) \quad (13) \]

Consequently $k$ must be negative, cosmos is so far proved infinite or open. And again, in virtue of
\[ T^{\alpha\beta}_{\gamma\delta} = (nU^\gamma)_\nu(U_{\mu}U^\mu)_\beta = 2U_{\mu}(U^\mu)_\beta = 2U^\mu(U_{\mu})_\beta = 0, \]
it follows that $d(\rho a^3) + pda^3 = 0$ and
\[ pd\left(\frac{1}{n}\right) + d\left(\frac{\rho}{n}\right) = 0 \quad (14) \]
Here $n$ represents the density of particle (galaxy) number. Since $\rho$ is assumed homogeneous, we may use the
weak field condition $p = -\rho$ proven above, and substituting it into Eq. (14) yields $d\rho = 0$, that is to say,
\[ \dot{p} = -\dot{\rho} = 0 \quad \text{or} \]
\[ p = -\rho = \text{const} = -\rho_0, \quad (15) \]
which is the most appropriate expression of energy conservation in infinite spacetime and indicates the singular
point of big bang did not exist. In addition, (14) implies the mass of galaxy changing with cosmic expansion since
\[ \rho/n \quad \text{stands for per particle mass.} \]
And further, the solution of Eq. (13), namely expanding factor, is given by
\[ a(t) = A \sin \left( t \sqrt{\frac{4\pi G \rho_0}{3}} \right). \quad (16) \]
Here $A$ is a positive constant. So far cosmic expansion and contraction are proved to be in circles like a
harmonic oscillator. (16) Means that the expansion of universe is decelerating and its contraction is accelerating,
this fact is compatible with the newest data observed, see figure 1 (Dominik J, 1993; Dai zi Gao, 2004; Fa Yin
Wang, 2009). We realize that the conclusion universe’s expansion is accelerating is wrong at all. In fact a
decelerating expansion is more acceptable for philosophy. We should be sobering that the accelerating universe is
not from direct measured data and instead it depends quite on cosmic model and if something is wrong with the
model the conclusion certainly fails.

Now we try to derive the relation between distance and red-shift. May as well put $a(t_0) = 1$, the light
from a galaxy to us satisfies (Weinberg, S. 1972) $1 + z = \frac{1}{a(t)}$ and $dz = -\frac{da}{a^2(t)}$. Here $z$ denotes red-shift. And
writing \[ \frac{4\pi G \rho_0}{3H_0^2} = q_0, \quad H(t_0) = H_0, \] we infer from Eq. (13)
\[
H \equiv \frac{da}{dt} = H_0 \sqrt{1 + q_0} (1 + z)^2 - q_0, \quad \text{and} \quad k = -H_0^2 (1 + q_0).
\]

Note that the subscript "0" refers to the present-day values. For the propagation of light line \( ds^2 = 0 \), then

\[
\frac{dt}{a(t)} = \frac{dz}{H} = -\frac{dl}{\sqrt{1 - kl^2}}, \quad \int_0^z \frac{dz}{H} = \int_0^l \frac{dl}{\sqrt{1 - kl^2}}. \quad l_a \text{ Denotes the galaxy's invariant coordinate. In view of}\]

luminosity-distance \( d_L = (1 + z) \int_0^l \frac{dl}{\sqrt{1 - kl^2}} \), we work out a new relation between distance and re-shift

\[
H_0 d_L = \frac{z + 1}{\sqrt{q_0 + 1}} \ln \left( \frac{(z + 1) \sqrt{q_0 + 1} + \sqrt{(q_0 + 1)(z + 1)^2 - q_0}}{1 + \sqrt{q_0 + 1}} \right)
\]

(17)

As \( z \to 0 \), expanding the right hand side of (17) into power series with respect of \( z \), (16) becomes

\[
H_0 d_L = z + \frac{1 - q_0}{2} z^2 + 3q_0^2 - 2q_0 - 1 + \frac{1}{6} z^3 + \ldots,
\]

which is the same result as that obtained via pure kinematics. The curved line in figure 1 (Dai zi Gao, 2004; Fa Yin Wang, 2009 ) is the image of (17) with \( q_0 = 0.14 \) and \( H_0 = 70 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1} \). The situation described by the curved line agrees well with the recent data of observations. Note that recent observations show that \( q_0 = \frac{4\pi G \rho}{3H_0^2} = \frac{\Omega_0}{2} = 0.1 \pm 0.05. \) ( Linder, E. V. 2003; Hamuy, M, 2003; Akaniz, J. S. 2004)

![Figure 1. The Recent Hubble diagram of 69 GRBs and 192 SNe Ia.](image)

Note that Distance-Modulus is equal to \( 5 \log d_L + 25 \), and the unit of \( d_L \) is Mpc.

Next we calculate "our" cosmic age, namely the time from last \( a(t) = 0 \) (at the moment, \( t \) may as well take 0 ) to today. Writing \( H(t_0) = H_0 \), from \( H = \frac{\dot{a}}{a} = 2\sqrt{\frac{\pi G \rho}{3}} \ctg \left( \frac{2\sqrt{\frac{\pi G \rho}{3}}}{t} \right) \), in the case that \( q_0 \) takes 0.14 "our" cosmic age is calculated as
$$t_0 = \frac{t_0^2}{H_0} \frac{q_0}{\sqrt{q_0}} = 1.37 \times 10^{10} a,$$

which agrees with observations. Besides, we can also compute how a galaxy's mass changes with time. Writing a galaxy' mass $m(t)$, taking account of $\rho = \text{const} = Nm(t)/a^3(t)$, where $N$ is equivalent to a proportional coefficient, immediately it is concluded that

$$\frac{m(t_1)}{a^3(t_1)} = \frac{m(t_2)}{a^3(t_2)},$$

which implies that galaxies can grow up without mergers and consists with recent observations (Genzel, R.2006). The formula (19) defines how a galaxy mass changes with evolution of universe.

And again, because any point can be thought the centre of universe's expansion, (19) can be looked as the rule of mass's change of any celestial body or galaxy. And applying (19) to the earth of today, we find that the increase of the earth's mass in a year is

$$\Delta m_0 = [\frac{a^3(t_0 + 1)}{a^3(t_0)} - 1]m(t_0) \approx 3H_0m_0 = 12.46 \times 10^{14} \text{kg}$$

And also deduce that the expanding speed of the radius of the earth is today $v_0 = H_0r_0 = 0.45 \text{mm/a}$.

By the way, from $a(t_0) = A \sin \left(\frac{t_0 \sqrt{4\pi G \rho_0}}{3}\right) = 1$ it can be decided that constant $A = \frac{1}{\sin \left(\frac{t_0 \sqrt{4\pi G \rho_0}}{3}\right)}$, and further we have the following relation of redshift $z$ and universe time $t$

$$1 + z = \frac{1}{a(t)} = \sin \left(\frac{t_0 \sqrt{4\pi G \rho_0}}{3}\right) / \sin \left(\frac{t \sqrt{4\pi G \rho_0}}{3}\right).$$

Here $t$ is the time at which photons was given out from the celestial body. The relation can be used to evaluate low limit of celestial body age.

We can also derive the density of galaxy number of any time $t$, Take $n_0$ for number density of galaxy of today $t_0$, and use proper speed $v_p = Hd_p$, where $d_p$ denotes proper distance of galaxy, then

$$dd_p = Hd_\rho dt, \text{ further } \frac{d_\rho}{d_{p_0}} = \exp \int_{t_0}^{t} Hdt,$$

and since galaxy number conserves, namely $n d_\rho^3 = n_0 d_{p_0}^3$, number density of galaxy of any time $t$ reads therefore

$$n = n_0 \exp \int_{t_0}^{t} 3Hdt = n_0 \left( \sin t_0 \sqrt{\frac{4\pi G \rho_0}{3}} / \sin t \sqrt{\frac{4\pi G \rho_0}{3}} \right)^{\frac{27}{4\pi G \rho}}$$

Which is the law that the density of galaxy number changes with time?

V. Exact interior solution of Eq. (12) and mechanism of celestial body's expansion.

In the case of static spherical symmetry, inside a celestial body (gravitational source), with $l$ as standard
radial coordinate the exact interior solution of Eq. (12) is given by.

\[ ds^2 = -\exp\left[C_2 + \int_{r_c}^{r} f(l) \left(1 + \frac{\omega(l)}{l}\right)^{-1} dl\right] dt^2 + \left(1 + \frac{G\omega(l)}{l}\right)^{-1} dl^2 + l^2 (d\theta^2 + \sin^2 \theta d\phi^2) \] (21)

in which \( \omega(l) = 4\pi \int_{l_0}^{l} \rho(l) l^2 dl, \quad f(l) = \frac{G}{l^3} \left[4\pi l^3 \rho(l) + \omega(l)\right], \quad l_c = l(r_c). \) Constant \( C_2 = \ln\left[1 - \frac{2GM}{l_c}\right], \) it makes sure \( g_{00} \) is continual on the boundary of the celestial body. Note that as scalar \( \rho = \rho(l) = \tilde{\rho}(r), \quad p = p(l) = \tilde{p}(r), \) and outside gravitational source both \( p \) and \( \rho \) vanish, namely \( \rho(l) = \tilde{\rho}(r) = \tilde{p}(r) = p(l) = 0 \) for \( r > r_c. \)

In order to determine the interior form of (21) in background coordinates, Eq. (4) is naturally extended as inside source

\[ \frac{dl}{dr} = \sqrt{1 + \frac{G\omega(l)}{l}} \exp\left[-G\int_{\xi}^{r} \rho(d'x') d'y' d'z'\right]. \] (22)

Obvious under the transformation of Eq. (22), line element (21) turns into

\[ ds^2 = -\exp\left[C_2 + \int_{r_c}^{r} f(l) \left(1 + \frac{\omega(l)}{l}\right)^{-1} dl\right] dt^2 + \exp\left[-2G\int_{\xi}^{r} \rho(d'x') d'y' d'z'\right] dr^2 + l^2 (d\theta^2 + \sin^2 \theta d\phi^2). \] (23)

Here \( l = l(r) \) is a specific function of \( r, \) which is determined by Eq. (22). Line element (23) is just the exact solution looked for and expressed in background coordinate system \( x^\mu = (x^0, x^1, x^2, x^3) = (t, r, \theta, \phi). \) Note that the solution of Eq. (22) satisfies the initial condition \( l(0) = 0. \) In fact, because there is no acceleration tendency for every direction at the centre gravitational source, \( dg_{00}/dr \) must be zero, and from (23) we have

\[ 0 = \frac{dg_{00}}{dr} = \frac{dl}{dr} \frac{dg_{00}}{dl} = \frac{dl}{dr} f(l) \left(1 + \frac{\omega(l)}{l}\right)^{-1} \exp\left[C_2 + \int_{r_c}^{r} f(l) \left(1 + \frac{\omega(l)}{l}\right)^{-1} dl\right], \]

which indicates \( f(l) = 0 \) at the centre, and so that \( l = l(0) = 0 \) at the centre. And if \( \rho = \text{const} = \frac{3M}{4\pi r_c^3}, \) then

\[ \int_{\xi}^{r} \rho(d'x') d'y' d'z' = \frac{3M}{2r_c} - \frac{M}{2r_c^3} r^3, \quad \omega(l) = 4\pi \int_{l_0}^{l} \rho(l) l^2 dl = \frac{M}{r_c^3} l^3, \] the solution of Eq. (22) is easily given by

\[ \sqrt{\frac{r_c}{GM}} \left[\ln\left(\sqrt{\frac{GM}{r_c^3}} + 1 + \frac{GM}{r_c^3} l^2\right) - l + \frac{GM}{6r_c} r^3 + \frac{1}{40} \left(\frac{GM}{r_c^3}\right)^2 r^5 + \ldots\right] \exp\left(-\frac{3GM}{2r_c}\right). \] (24)

Though energy density \( \rho, \) generally speaking, isn't a constant, we may take its average value or piecewise integrate on \( r \) in practice for the convenience of calculation. As an important example, on the surface of the Sun
\[ r = r_e = 6.96 \times 10^8 \text{ m, } M = 1.99 \times 10^{30} \text{ kg, using (24), that is taking average value of } \rho, \text{ we can work out the surface's } l = l(r_e) = 6.96 \times 10^8 \text{ m} - 1720 \text{ m}, \text{ which is highly equal to the Sun's radius. And likewise, we can work out } l = 6371 \text{km} - 0.00038 \text{km} \text{ on the Earth's surface, and this almost equals the Earth's radius 6371km.} \]

So far, using the continuity of \( l = l(r) \) not only we can determine the constant \( C_i \) but also can calculate the deflected angle of light line on the surface of Sun. For photon's propagation outside Sun from (6) we have

\[
0 = ds^2 = -\left(1 - \frac{2GM}{l}\right)dt^2 + \exp\left(-\frac{2GM}{r}\right)dr^2 + l^2 \left(d\theta^2 + \sin^2 \theta d\varphi^2\right)
\]

\[
= -\left(1 - \frac{2GM}{l}\right)dt^2 + \left(1 - \frac{2GM}{l}\right)^{-1} dl^2 + \left(d\theta^2 + \sin^2 \theta d\varphi^2\right)l^2.
\]

Similar to former calculation, the deflected angle is given by \( \alpha = \frac{4MG}{l} = \frac{4MG}{l(r_e)} = 1.78^\circ \), which is more consistent with observational result (1.89\(^\circ\)) compared with former theoretical value \( \alpha = \frac{4MG}{r} = \frac{4MG}{r_e} = 1.75^\circ \).

On the other hand, the conserved law gives

\[
\frac{dp}{dl} = G(p + \rho)p + \frac{\omega}{2}(l^2 + lG\omega(l))^{-1}.
\]

On the boundary the gravity acceleration should be continual, according to (2), using (4), (6), (22), (23) we have

\[ (\Gamma_{00})_{r=r_e^+} = (\Gamma_{00})_{r=r_e^-}, \text{ that is, (} g^{11} \frac{dg_{00}}{dr} \bigr)_{r=r_e^+} = (g^{11} \frac{dg_{00}}{dr} \bigr)_{r=r_e^-}, \text{ it follows that}
\]

\[
\left[ \frac{dl}{dr} \frac{d}{dl} \left(1 - \frac{2GM}{l}\right) \right]_{r=r_e^+} = \left[ \frac{dl}{dr} \frac{d}{dl} \exp \left[ C_2 + \int_{l_e}^l (1 + \frac{\omega(l)}{l})^{-1} dl \right] \right]_{r=r_e^-}
\]

And after simplifying further, it becomes

\[
[4\pi l_e^3 p + \omega(l_e)]\sqrt{l_e - 2GM} = -2M\sqrt{l_e + G\omega(l_e)},
\]

which is the boundary condition \( p \) must satisfy, and the condition defines \( p \) to be negative within celestial body.

For general cases, inside source, gravitational field is still which means \( l = l(r) \approx r, \frac{2GM}{r} << 1 \), and from (26) the boundary pressure \( p \approx -\frac{3M}{4\pi r_e^3} = -\bar{\rho} \), which is consistent with (11). Here \( \bar{\rho} \) denotes average. As an emphasis, we must point out that when (1) or (6) is applied to a mass point of the surface of the static source, it exists that \( 0 \geq ds^2 = -(1 - \frac{2GM}{l})dt^2 \), which indicates that \( 1 - \frac{2GM}{l} \) of static source is nonnegative.

Next let us weak, consider a small volume \( V_i \) of mass \( m_i \) inside source, \( dV_i \) denotes \( V_i \)'s change caused from
the Expansion of space-time, in view of Eq. (12) we have \( dm_i = - p_i dV_i \), hence

\[
d p_i = d\left( \frac{m_i}{V_i} \right) = - \left( \rho_i + p_i \right) \frac{dV_i}{V_i} = - \left( \rho_i + p_i \right) \frac{d^3(t)}{a^3(t)},
\]
which means that for arbitrary point it holds that

\[
\frac{\partial \rho}{\partial t} = - \frac{\rho + p}{a^3(t)} \frac{d^3(t)}{dt},
\]
(27) determines how matter density changes locally. It is seen from (27) that when celestial bodies expand with cosmic expansion its density may be unchanging in the case of \( \rho + p = 0 \). So far, we deduce that bursts of celestial bodies and formation of earthquakes originate both from unceasing accumulation of inside matter and change of distribution; and it is the negative pressure that gets matter in celestial body continuously produce. (Nashed G.G.L, 2011)

VI. Cracking of the puzzle of dark matter.

The negative pressure as important gravitational source is invisible, and it is the negative pressure that appears as the form of dark matter and leads to the phenomenon of missing mass, or say that so-called dark matter is just the negative pressure, this fact are showed as follows.

Speaking generally, within a galaxy the metric field is weak field, and when a galaxy is treated as a celestial body of spherical symmetry, according to the discussion in section III, within the galaxy \( 0 \leq r \leq r_e \) pressure \( p = \text{const} \neq 0 \). And from (11) we infer \( p = \text{const} = -\frac{3M}{4\pi r_e^3} \), and further we have

\[
h_{00} = -G \int \frac{D + 3p}{\xi} dx' dy' dz' = -4\pi G \left( r^{-1} \int_0^r \rho r^2 dr + \int_0^{r_e} \rho r dr - \int_0^r \rho dr \right) - 6G\pi pr_e^2 + 2G\pi pr^2
\]

According to (2) the gravity acceleration (or gravitational field strength) within the galaxy is given by

\[
g = -\Gamma_{00} = \frac{1}{2} \frac{dh_{00}}{dr} = 2\pi Gpr + \frac{2\pi G}{r^2} \int_0^r \rho r^2 dr = 2\pi Gpr + \frac{Gm(r)}{2r^2}
\]

where \( m(r) \equiv 4\pi \int_0^r \rho r^2 dr \), and \( g \) may be positive or negative since pressure is negative, and the negative \( g \) indicates the direction of acceleration is towards centre. And according to (2) the corresponding round orbital speed \( v_r \) is given by

\[
v_r^2 = -gr = -2\pi Gpr^2 - \frac{Gm(r)}{2r},
\]
(28)

From (28) it is seen that when \( m(r) \) looks even on the verge of zero near the centre of the galaxy the speed \( v \) can become high too, and this explains the phenomenon of so-called missing mass. Again, from (28) we get

\[
2rv_r^2 = -4\pi Gpr^3 - Gm(r), \quad \text{if} \quad v \quad \text{is a constant between} \quad r_1 \quad \text{and} \quad r_2, \quad \text{differentiating this equation and} \quad
\]
using \( v_T^2 (r_1 \leq r \leq r_2) = -2\pi Gpr^2 - \frac{Gm(r)}{2r} = \frac{3MG}{2r_1^3} r_2^2 - \frac{Gm(r)}{2r_1} \) yield

\[ \rho(r_1 \leq r \leq r_2) = -3p - \frac{v_T^2}{2\pi G r^2} = \frac{9M}{4\pi r_e^3} - \frac{3M}{4\pi r_e^3} r_1^2 + \frac{m(r)}{4\pi r_e^2} \]  

which is the condition a typical spiral galaxy with a halo satisfies. May as well set \( r_1 = nr_2 \) ( \( 0 < n < 1 \)), then

\[ m(r_2) = m(r_1) + 4\pi \int_{r_1}^{r_2} \rho r^2 dr = \frac{3M}{r_e^3} (1 - n^2) r_2^3 + \frac{m(r_1)}{n} \]

and in consideration of \( 0 \leq m(r_2) \leq M \) we concluded that

\[ 0 \leq r_2^3 \leq \frac{nM - m(r_1)}{3nM (1 - n^2)} r_e^3 \]  

Which indicates it is impossible for \( r_2 \) to arrive at the galaxy’s edge \( r_e \) in the case of \( n < \sqrt{2/3} \). Obviously, if \( \rho \) begins to decrease from \( r_2 \) to \( r_e \) both \( v_T \) and \( |g| \) begin to increase. Of course, it isn't easy to observe the speed of the particles between \( r_2 \) and \( r_e \) because near the edge \( r_e \) matter becomes virtually very thin. The curve in figure 2 describes the situation predicted by (28) and (30), and it is in conformity with recent observational results (Cayrel, R. 2001). 

\[ \text{Figure 2. The velocity distribution diagram} \]

So far, we conclude that so-called dark matter is just the effect of the negative pressure or say that the negative pressure is just so-called dark matter; and the dark matter (Genzel, R. 2006; Baojiu Li, 2008) puzzle has naturally been cracked. Of course, so-called dark energy problem is also removed since cosmological constant is reconfirmed as zero again and the concept of dark energy becomes unnecessary in the new amendme
VII. Motion in centre field and formation of galaxies and background photons

Equation (15) indicates that not only space is expanding but also celestial bodies or galaxies themselves, that is, like a expanding balloon, the ink prints on it also expand at the same proportion. This is just the elementary mechanism of galaxy formation. In order to illuminate galaxy formation clearer we look into the motion in centre field. Let \( M \) denote mass of centre body. Generally speaking, its gravitational field is weak, geodesic reduces to Newton’s law, for a object moving around the centre body we have

\[
\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r},
\]

(31)

where \( r \) is the radius of round orbit, \( T \) is revolution period. Noticing \( M \) to be variable now and to satisfy (19) and using (31) we infer

\[
\Delta r = r(1 + \Delta T/T)^2 \left[ a(t + \Delta t)/a(t) - 1 \right] \text{ from } t \text{ to } t + \Delta t.
\]

And putting \( \Delta t \to 0 \) we have

\[
v \equiv \frac{dr}{dt} = rH + \frac{2r}{3T} \frac{dT}{dt}.
\]

(32)

where the final term is explained as perturbation and gravitational radiation. For instance, apply (32) to the motion round today’s Earth, for geostationary satellite, neglecting perturbation and gravitational radiation, namely taking \( dT_0 = 0 \) we find that its orbit radius will increase by \( \Delta r_0 = 2.7 \text{ mm} \) in a year. And for the motion of Moon, observations show that its orbit radius increases by 0.38cm in a year today, then using (32) we conclude that the orbit period \( T_0 \) of Moon will slow by 0.0001s in a year today.

When (32) is used to the edge of a spiral galaxy, it is concluded that the terminuses of spiral arms gradually stretch outward. Of course, other points near the terminuses continuously follow and form involutes. See the following figure 3.

![Expanding centre and Gradually stretching out arm](image)

Figure 3: sketch map of formation and evolution of spiral arms

Eq. (32) means that separating speed from centre lies on \( v = rH \) neglecting perturbation and radiation damp.

It is important to realize that the spin of a system is the composition of orbit motion of many particles, spin and orbit motion do not have essential difference. And for celestial body’s expansion, lying on \( v = rH \) means its spin period not to change.
Note that the existence of Eq. (32) doesn’t mean the destruction of conservation of angular momentum on whole because mass \( M \) is connected with the factor \( a(t) \), which embodies the interaction among galaxies, the nonconservation of angular momentum of individual galaxy is admitted.

Again, the fact that space, celestial bodies and galaxies simultaneously expand proportionally links the homogeneity of today’s universe in a large range with that of early universe in a small range, because the large range is just the amplification of early the small range. Background radiation has proven early universe to be homogeneous in quite small range. Therefore our conclusion is in accordance with observations.

The following figure 4 is the global picture of galaxy evolution and distribution under \( \rho = const \) in different time stages, the earlier, the smaller and the denser. Figure 5 is the picture of galaxies seen by today’s telescope, and the farther, the earlier and the evener.

Note that the horizon at moment \( t > 0 \) is now according to (16)

\[
d\hat{h}(t) \equiv \frac{1}{a(t)} \frac{dt}{\sqrt{\frac{4\pi G \rho_0}{3}} } = \frac{dt}{\sin \left( t \sqrt{\frac{4\pi G \rho_0}{3}} \right) } = \infty
\]

That is to say, so-called horizon puzzle or homogeneity puzzle does not exist in the present theory framework at all, and need not introduce inflation like a patch on theory.

Naturally, the microwave background radiation measured today is the compositive effect of various photons emitted by innumerable galaxies remote, whose distances to us are unidentifiable, which comprised infinitely deep thin gas and could absorb any frequency photon and therefore possess black body feature.

Note that the state that horizon vanishes is unobservable though \( d\hat{h}(t) = 0 \) for \( t = 0 \), because any observation carried out needs a time lag \( \Delta t \)

![Figure 4. The global picture of galaxy evolution and distribution in different stages](image-url)
Figure 5: The actual picture of galaxies seen by today’s telescope

Now, referring to figure 5 we try to solve all-direct galaxy number \( dN_G \) between \( z \to z + dz \), which is an observational quantity for our telescope today. From the discussion above we know proper distance of galaxies of redshift \( z \) is given by

\[
d_p = \int_0^z \frac{dz}{H} = \frac{1}{H_0 \sqrt{q_0 + 1}} \ln \left( \frac{(z + 1)\sqrt{q_0 + 1} + \sqrt{(q_0 + 1)(z + 1)^2 - q_0}}{1 + \sqrt{q_0 + 1}} \right),
\]

where \( H = H_0 \sqrt{(1 + q_0)(1 + z)^2 - q_0} \), and number density of galaxies near proper distance \( d_p \) or redshift \( z \)

reads

\[
n = n_0 \exp \int t_0^t 3Hdt = n_0 \exp \int_0^z \frac{3}{1 + z} dz = n_0 (1 + z)^3,
\]

where \( z \) is redshift of galaxies near proper distance \( d_p \), we easily obtain the following result

\[
dN_G \equiv 4 \pi n_0 (1 + z)^3 \ln \left( \frac{(z + 1)\sqrt{q_0 + 1} + \sqrt{(q_0 + 1)(z + 1)^2 - q_0}}{1 + \sqrt{q_0 + 1}} \right) dz,
\]

where \( n_0 \) take the value of number density of the galaxies around us.

Finally, we prove that the dark body spectrum of background photons keep on in the course of the propagation. Assume that background photons arrive at B from A between \( t = t_A \) and \( t = t_B \). See the following figure 6.

Figure 6: the travelling background photons toward us.
Take $\sigma(\nu, t) d\nu$ for background photon’s number density between frequency $\nu \rightarrow \nu + d\nu$ at the time $t$. Assume that photon number is conserved in the course of propagation, then we have

$$a^3(t_A)\sigma(\nu_A, t_A) d\nu_A = a^3(t_B)\sigma(\nu_B, t_B) d\nu_B$$

Take $T_A$ for the temperature of background photons at the time $t = t_A$, according to Boltzmann statistics law, the average kinetic energy of photons equals $\frac{3}{2}T$ (here Boltzmann constant takes 1)

$$T_A = \frac{2\int_0^\infty a^3(t_A) h\nu \sigma(\nu_A, t_A) d\nu_A}{\int_0^\infty a^3(t_A) \sigma(\nu_A, t_A) d\nu_A},$$

and based on the same reason $T_B = \frac{2\int_0^\infty a^3(t_B) h\nu \sigma(\nu_B, t_B) d\nu_B}{\int_0^\infty a^3(t_B) \sigma(\nu_B, t_B) d\nu_B}$.

Note that $h\nu$ represents kinetic energy of a photon of frequency $\nu$. And using reshift relation $\nu_A(t_A) = \nu_B(t_B)$, where $\nu_A$ and $\nu_B$ are the frequency of the same photon at the time $t = t_A$ and $t = t_B$, it is proven that

$$a(t_A)T_A = a(t_A) \cdot \frac{2\int_0^\infty a^3(t_A) h\nu \sigma(\nu_A, t_A) d\nu_A}{\int_0^\infty a^3(t_A) \sigma(\nu_A, t_A) d\nu_A} = \frac{2a(t_B)\int_0^\infty a^3(t_B) h\nu \sigma(\nu_B, t_B) d\nu_B}{\int_0^\infty a^3(t_B) \sigma(\nu_B, t_B) d\nu_B} = T_B a(t_B)$$

Where $T_B$ denotes the temperature of background photons at the time $t = t_B$. If background photons at position A satisfy black body spectrum, that is to say, $\sigma(\nu_A, t_A) d\nu_A = \frac{8\pi\nu_A^3 d\nu_A}{\exp\frac{h\nu_A}{T_A} - 1}$ (Planck law), then using $\nu_A a(t_A) = \nu_B a(t_B)$, namely $a(t_A) d\nu_A = a(t_B) d\nu_B$, we obtain

$$\sigma(\nu_B, t_B) d\nu_B = \frac{\sigma(\nu_A, t_A) d\nu_A a^3(t_A)}{a^3(t_B)} = \frac{8\pi\nu_A^3 d\nu_A}{\exp\frac{h\nu_A}{T_A} - 1} \frac{a^3(t_A)}{a^3(t_B)} = \frac{8\pi\nu_B^3 d\nu_B}{\exp\frac{h\nu_B}{T_B} - 1}$$

So far our conclusion has been proven. Note that if $t_A = 0$ , $a(0) = 0$ , then any $\nu_A \rightarrow \nu_B = 0$ , which implies that the background photons to come to us were given off in different time, and the photons whose re-shift are bigger were given off in earlier time and from farther source. As a result, we get the conclusion that the lower frequency of background photons measured today is, the smaller their density fluctuation or anisotropy is which is in accordance with recent observation. This property of cosmic background radiation indicates just big bang did not exist because the background photons measured today were never given off by the so-called final scattering surface that big bang implied, if they were so, the density fluctuation or anisotropy of different frequency’s background photons should be consistent or synchronous and should not have thing
to do with frequency as matter distribution on the final surface is definite at the same time and the information's carried by different frequency’s background photons react to the same surface’ situation of matter distribution, but fact is opposite. That is to say, cosmic background radiation is never they remain of so-called big bang.

Though the temperature of background photons become lower and lower due to their re-shift, the average temperature of cosmic matter should be unchanged all along, the singular point of big bang should not exist. In the new theory cosmic temperature keeps unchanged since cosmic energy density is proven unchanged, all observations can parallel and even better be explained. The temperature of background photons observed today never represents that of universe today, it is worthy of laying stress that the temperature of universe means the average of all matter’s temperature but not only background photons’ temperature, and talking about temperature leaving matter has no meaning. In principle, cosmic temperature can be measured directly, we may suppose that the temperature of the Milky Way represents that of universe because it is a moderate galaxy, that is to say, the temperance of universe is far higher than that of background photons measured today.

VIII. Quantum process of continuous creation of matter in celestial bodies

\( P = -\rho \) Tells us that the negative pressure in celestial bodies is actually a negative energy field, and there \( p \) and \( \rho \) excite with each other and generate simultaneously. Connecting with particle physics it is naturally deduced that in celestial bodies many particle-antiparticle pairs (including neutron and antineutron, proton and antiproton, electron and positron and so on) can ceaselessly produce and annihilate, the antiparticles lie in negative energy level------can't be observed, the particles lie in positive energy level, and the absolute value of energy of particle and antiparticle is equal. Let \( \Delta t \) denote the lifetime of a kind of particle-antiparticle pairs, namely the average time from production to annihilation, according to uncertain principle the range \( \Delta E \) of energy satisfies

\[
\Delta E \geq \frac{\hbar}{2\Delta t},
\]

which shows that instantaneous energy of new particle may be very high. Note that not all of the particles annihilate as soon as they come into being, only those which don't not have opportunity in the time \( \Delta t \) to react with the surrounding particles or to collide and change their energy can annihilate, once the reaction with other particles or the collision occur the annihilation no longer carry out, and in this case the negative energy field detains a negative energy antiparticle while the particle becomes constituents of matter. Therefore, the negative energy field is too a quantum field to consist of various negative energy antiparticles. Of course, an antiparticle of energy \( -\varepsilon \) can be excited to energy \( \varepsilon \) by a meson of energy \( 2\varepsilon \) and becomes the antiparticle that can be observed. For no other reason than that many antiparticles lie in negative energy level and can't be observed, we perceive that particles and antiparticles aren't symmetrical. As a result of general relativity, Eq. (15) in section IV exposes already that matter and antimatter are symmetrical.

Obviously the negative pressure field, not only thermal nuclear reactions, provides energy source of star radiation, therefore the mystery of solar neutrino doesn't exist in the new theory framework.

And considering of tunneling effect in quantum theory, many nuclear reactions are able to complete slowly in celestial bodies even if the temperature ( average kinetic energy of particles ) is low, which implies that in the case of low temperature elements can also compose. As for what kind of nuclear reaction is in evidence, this depends on temperature of celestial bodies. And as a result, the abundance of elements in a celestial body is the effect of various nuclear reaction for long time.

For a celestial body of temperature \( T \), we may as well treat all atoms in it as a open thermodynamic system, whose giant distribution function according to quantum statistics is given by
\[ \rho = \exp(-\Psi - \sum_{i=1}^{k} \alpha_i N_i - \beta E) \]

Where \( N_i \) denotes the number of atoms of \( i \)-th kind element. And let \( m_i \) denote its mass, the total energy \( E = \sum_{i=1}^{k} N_i m_i \), then the average value of atom number of element of \( j \)-th kind reads

\[
\bar{N}_j = \frac{\sum_{N_1=0}^{\infty} \cdots \sum_{N_k=0}^{\infty} \exp N_j \left[ -\nu - \sum_{i=1}^{k} N_i (\alpha_i + \beta m_i) \right]}{\sum_{N_1=0}^{\infty} \cdots \sum_{N_k=0}^{\infty} \exp \left[ -\nu - \sum_{i=1}^{k} N_i (\alpha_i + \beta m_i) \right]} = -\frac{1}{m_j} \frac{\partial}{\partial \beta} \ln \left( \sum_{N_j=0}^{\infty} \exp(-\alpha_j - \beta m_j)N_j \right) = \frac{1}{m_j} \frac{\partial}{\partial \beta} \ln(1 - e^{-\alpha_j - \beta m_j}) = \frac{1}{\exp(\alpha_j + \beta m_j) - 1} = \frac{1}{\exp(m_j - \mu_j) - 1}
\]

Here \( \mu_i \) amounts to the chemical potential of the group, \( T \) is the temperature of the celestial body, namely average kinetic energy of all atoms, \( k \) is Boltzmann constant. From above relation we have for arbitrary two elements \( A \) and \( B \)

\[
\frac{\bar{N}_A}{\bar{N}_B} = \frac{\exp(m_B - \mu_B) - 1}{\exp(m_A - \mu_A) - 1}
\]  \((34)\)

which decides the abundance of elements in a celestial body. Observations of astronomy show that element abundance is different in different celestial bodies, which is consistent with \((34)\). Observations of astronomy show that the abundance of elements is in accordance seen from large scope, which implies both temperature and chemical potential are uniform seen from large scope. Observations of astronomy show that all elements in other celestial bodies can also be found out on the earth, which implies that the origin of various elements is in the same way, namely they originate all production and annihilation of particle-antiparticle pairs.

**IX. Conclusions:** Density and pressure of universe do not change all along (Massimiliano, 2001), the singularity of big bang didn’t exist (Mei Xiaochun, 2011) and matter in universe is produced continuously and slowly. With cosmic expansion celestial bodies and galaxies expand too, which is just the fundamental mechanism of celestial body or galaxy formation. The dark matter to appear as negative pressure is just the antimatter that lies in negative energy level and thus cannot be observed, which cannot exist alone and must be accompanied by ordinary matter.

**X. Appendices.**

**A:** the derivations of \((21)\) and \((25)\)

According to description of general relativity, in the case of static spherical symmetry, in standard coordinate system the form of invariant line element is written as

\[
ds^2 = -d\tau^2 = B(\ell) dt^2 + A(\ell) \ell^2 + \ell^2 (d\theta^2 + \sin^2 \theta d\varphi^2),
\]
Where \( l \) is called standard radial coordinate, space-time coordinate \( x^\mu = (x^0, x^1, x^2, x^3) = (t, l, \theta, \varphi) \).

\[
g_{00} = g_{ll} = -B(l), \quad g_{11} = g_{rr} = A(l), \quad g_{22} = g_{\theta\theta} = l^2, \quad g_{33} = g_{\varphi\varphi} = l^2 \sin^2 \theta, \quad \text{the other components}
\]
equal zero. From the definition of inverse Matrix we work out

\[
g^{00} = -\frac{1}{B}, \quad g^{11} = \frac{1}{A}, \quad g^{22} = \frac{1}{l^2}, \quad g^{33} = \frac{1}{l^2 \sin^2 \theta}, \quad \text{the others equal zero.}
\]

And from connection \( \Gamma_\mu^\nu = \frac{1}{2} g^{\sigma \rho} \left( \frac{\partial g_{\nu \rho}}{\partial x^\sigma} + \frac{\partial g_{\sigma \rho}}{\partial x^\nu} - \frac{\partial g_{\nu \sigma}}{\partial x^\rho} \right) \), we work out

\[
\Gamma_1^1 = \frac{A'}{2A}, \quad \Gamma_2^2 = -\sin \theta \cos \theta, \quad \Gamma_3^3 = \cot \theta, \quad \Gamma_0^0 = -\frac{l}{A} \sin^2 \theta, \quad \Gamma_1^3 = \Gamma_3^1 = \frac{1}{l},
\]

the others are zero, where \( A' = \frac{dA}{dl}, \quad B' = \frac{dB}{dl} \).

And from connection

\[
\Gamma_\eta^\mu = \Gamma_\mu^\eta = -\Gamma_\nu^\eta + \Gamma_\nu^\sigma \Gamma_\sigma^\eta - \Gamma_\sigma^\nu \Gamma_\sigma^\eta, \quad \text{we work out}
\]

\[
R_{\nu \sigma} = R_{\sigma \nu} = -\Gamma_\mu^\nu \Gamma_\sigma^\mu + \Gamma_\mu^\sigma \Gamma_\eta^\nu - \Gamma_\eta^\mu \Gamma_\eta^\sigma, \quad \text{the others are zero.}
\]

On the other hand

\[
T_{\mu \nu} = (\rho + p) U^\mu U^\nu + pg_{\mu \nu}, \quad g^{\mu \nu} U^\mu U^\nu = -1, \quad T = g^{\mu \nu} T_{\mu \nu} = 3p - \rho,
\]

and for the case of static spherical symmetry \( p = p(l), \quad \rho = \rho(l), \quad U_0 = -\sqrt{B}, \quad U_i = 0 \), then we work out

\[
T_{00} - \frac{T}{2} g_{00} = \frac{B(3p + \rho)}{2}, \quad T_{33} - \frac{T}{2} g_{33} = l^2 \sin^2 \theta \frac{(\rho - p)}{2}, \quad T_{22} - \frac{T}{2} g_{22} = \frac{l^2 (\rho - p)}{2},
\]

\[
T_{11} - \frac{T}{2} g_{11} = \frac{A(\rho - p)}{2}
\]

the other corresponding components are zero. Field equation (12) is equivalent to

\[
R_{\mu \nu} = 4\pi G (T_{\mu \nu} - \frac{1}{2} T g_{\mu \nu}), \quad \text{we get the following three independent equations:}
\]

\[
\begin{align*}
R_{00} &= -\frac{B''}{2A} + \frac{B'}{4A} \left( \frac{A'}{A} + \frac{B'}{B} \right) - \frac{B'}{l A'} = 2\pi G (\rho + 3p) B \\
R_{11} &= \frac{B''}{2B} - \frac{B'}{4B} \left( \frac{A'}{A} + \frac{B'}{B} \right) - \frac{A'}{l A} = 2\pi G (\rho - p) A \\
R_{22} &= \frac{l}{2A} \left( \frac{A'}{A} + \frac{B'}{B} \right) + \frac{1}{A} - 1 = 2\pi G (\rho - p) l^2
\end{align*}
\]

And the other corresponding equations are identities. Then we have

\[
\frac{R_{00}}{2B} + \frac{R_{11}}{2A} + \frac{R_{22}}{l^2} = \frac{1}{l^2} + \frac{1}{A l^2} \frac{A'}{A l} = 4\pi G \rho, \quad \text{namely} \quad \left( \frac{1}{A} \right)' = 1 + 4\pi G \rho l^2,
\]

And since \( A(0) \) is limited, we infer

\[
A(l) = \left( 1 + \frac{G \omega(l)}{l} \right)^{-1}, \quad \text{where} \quad \omega(l) = 4\pi \int_0^l \rho(l) l^2 dl.
\]

On the other hand, the conservation law \( T_{\mu \nu}^\sigma = 0 \) gives

\[
\frac{B'}{B} = -\frac{2p'}{p + p}, \quad \text{then from}
\]

...
\[ R_{22} = \frac{l}{2} + \frac{G\omega(l)}{l} \left[ \frac{G}{l^2} - \left( \omega + \omega \right) \left( 1 + G\omega \right)^{-1} - \frac{2p}{\rho + p} \right] + \left( 1 + G\omega \right)^{-1} = 2\pi G(\rho - p)l^2, \] after being simplified

\[ \frac{dp}{dl} = G\left( p + \rho \right) \left( 2\pi l^3 p + \frac{\omega}{2} \right) \left( l^2 + lG\omega(l) \right)^{-1}. \]

And again, from \( B' \) \( B = -\frac{2p}{\rho + p} = -2G\left( 2\pi l^3 p + \frac{\omega}{2} \right) \left( l^2 + lG\omega(l) \right)^{-1}, \) we obtain

\[ B(l) = \exp \left[ C_2 + \int_{l'}^{l} f(l) \left( 1 + \frac{\omega(l)}{l} \right)^{-1} dl \right], \]

Where \( f(l) = \frac{G}{l^2} \left[ 4\pi l^3 p(l) + \omega(l) \right], \) and constant \( C_2 = \ln \left( 1 - \frac{2GM}{l_c} \right), \) it makes sure \( B(l) \) is continuous on the bound \( r_e \) (surface of source). Note that the value of \( I(r_e) \) on the bound is determined by (24).

**B:** the derivation of luminosity distance \( d_L = \left( 1 + Z \right)^{\frac{l}{l}} \frac{dl}{\sqrt{1 - kl^2}}. \)

At the moment \( t \) proper distance of a galaxy is defined as \( d_p = a(t) \int_0^{l(t)} \frac{dl}{\sqrt{1 - kl^2}}. \) Let a telescope of area \( A \) faces the galaxy. Within time \( \delta t_e \) the galaxy emitted \( n \) photons of total energy \( nh\nu_e, \) and within time \( \delta t_0 \), they arrive at the telescope. Spectrum radiate power of galaxy is defined as \( L \equiv \frac{nh\nu_e}{\delta t_e}. \) Power received by telescope is \( p = \frac{nh\nu_0}{\delta t_0} \frac{A}{4\pi d_p^2(t_0)} \). Using \( \nu_0 = \frac{\nu_e a(t_e)}{a(t_0)} \) and \( \frac{1}{\delta t_0} = \frac{a(t_e)}{\delta t_e a(t_0)}, \) we have

\[ p = \frac{nh\nu_0}{\delta t_0} \frac{A}{4\pi d_p^2(t_0)} = \frac{nh\nu_e a^2(t_e)}{\delta t_e a_0^2(t_0)} \frac{A}{4\pi a^2(t_0)d_p^2(t_0)} = \frac{La^2(t_e)}{4\pi a^2(t_0)d_p^2(t_0)}. \]

Vision luminosity received by telescope is defined as \( l \equiv \frac{p}{A} = \frac{La^2(t_e)}{4\pi a^2(t_0)d_p^2(t_0)}. \) We know that vision luminosity of light source in Euclidean space is \( l = \frac{L}{4\pi d^2}, \) and generalizing the definition to curve space,

luminosity distance \( d_L = \sqrt{\frac{L}{4\pi l}} = \frac{a(t_0)}{a(t_e)} d_p(t_0) \). Using \( \frac{a(t_0)}{a(t_e)} = 1 + Z \) and putting \( a(t_0) = 1, \) finally we obtain

\[ d_L = (1 + Z)d_p(t_0) = \left( 1 + Z \right)^\frac{l}{l} \frac{dl}{\sqrt{1 - kl^2}}. \]
References


