Radar Cross Section of Exhaust Ions in HF Region

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Abstract

We review the radar cross section (RCS) of exhaust particles and its effect in the RCS of aerial targets in HF region. The results show that the plume of rockets and aircraft engine can increase the RCS. This effect can explain by the production and concentration of electrons and positive / negative ions at the exhaust. According to experimental and simulation results we can add the RCS of exhaust to the target RCS and in HF frequencies we can treat the engine plasma as perfect electric conductor.

Keywords: radar cross section, exhaust ionization, HF frequency.

Introduction

In the high-frequency band (3-30 MHz), the radar cross section of an ascending rocket results from the reflection of radio waves off both the body of the rocket and the adjacent exhaust stream of partially ionized products of the rocket fuel combustion [1]. The electromagnetic fields scattered from the exhaust stream and from the ballistic missile body are incoherent, because the boundaries, structure, and dimensions of the flame change continuously during flight. As a result, the phase of the radio waves scattered from the exhaust flame does not exhibit a constant relation to the phase of the wave reflected from the missile body. The geometric dimensions of the main elements of the exhaust stream may be estimated with the empirical relations presented in [1]. The shape of the core of the stream of combustion products is well approximated by an ellipse. Therefore, the RCS of the nucleus of the flame may be calculated from the formula for the RCS of a circular metallic ellipse [2,3]:

\[ \sigma_\perp = \frac{16 \pi k_0^4}{9} \left[ \frac{\sin^2(\phi)}{I_a} + \frac{\cos^2(\phi)}{I_b} + \frac{\cos^2(\phi)\cos^2(\theta)}{I_b + I_c} + \frac{\sin^2(\phi)\cos^2(\theta)}{I_a + I_c} + \frac{\sin^2(\theta)}{I_b + I_a} \right] \]  

\[ \sigma_\parallel = \frac{16 \pi k_0^4}{9} \left[ \frac{\cos^2(\phi)\cos^2(\theta)}{I_a} + \frac{\sin^2(\phi)\cos^2(\theta)}{I_b} + \frac{\sin^2(\theta)}{I_c} + \frac{\sin^2(\phi)}{I_b + I_c} + \frac{\cos^2(\phi)}{I_a + I_c} \right] \]  

Where the \( I_a, I_b, I_c \) are:

\[ I_a = \int_0^\infty \frac{dv}{(a^2 + v)((a^2 + v)(b^2 + v)(c^2 + v))^\frac{1}{2}} \], \quad \[ I_b = \int_0^\infty \frac{dv}{(b^2 + v)((a^2 + v)(b^2 + v)(c^2 + v))^\frac{1}{2}} \], \quad \[ I_c = \int_0^\infty \frac{dv}{(c^2 + v)((a^2 + v)(b^2 + v)(c^2 + v))^\frac{1}{2}} \].

We show that small variations at the surface of target don’t modify the RCS of target. The properties of absorbing materials are extremely depending on frequency and their response is acceptable at small wavelength domain. Therefore, in HF frequencies the shaping is not important, and the low observable objects (LO) can be detected. Because we work at Rayleigh region, the main scattering process is forward scattering and by a bi--static or multi--static HF radar

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If we vary the surface of scatterer, then we can write the new surface integral as:

$$\vec{H}^s = \frac{1}{4\pi} \int \left[ (\vec{n}' \times \vec{H}^T (\vec{r})) \times \vec{\nabla}' \psi_0 \right] ds'.$$  \hfill (3)

If we use Taylor expansion for $H$ filed on the surface, we obtain the following relations for scattered field:

$$\vec{H}^s = \frac{1}{4\pi} \int \left[ (\vec{n}' \times \vec{H}^T (\vec{r} + \delta \vec{r})) \times \vec{\nabla}' \psi_0 \right] ds' + \frac{1}{4\pi} \int \left[ (\vec{n}' \times \vec{H}^T) \times \vec{\nabla}' \psi_0 \right] ds' \hfill (4)$$

$$+ \frac{1}{4\pi} \int (\vec{n}' \times \vec{H}^T) \times \vec{\nabla}' \psi_0 \right] ds'.$$

1. Scattering Formalism and Approximation in Rayleigh Region

The far field approximation of scattered field from PEC is given in standard EM textbooks. We use the following relation for scattered field $[2,3]$

$$\vec{H}^s = \frac{1}{4\pi} \int [\vec{n}' \times \vec{H}^T (\vec{r})] \times \vec{\nabla}' \psi_0 \right] ds'.$$  \hfill (5)

We can ignore the second and third terms, if $\delta r \ll \lambda$. This means that if the wavelength of incident EM wave is large in compare of surface variation, then the scattering cross section is independent of the surface variation. Therefore, the small variations in shape of target are not important in Rayleigh region. The RCS of a civil air craft with simplified model and analytical results for spheroid scattered approximation are given in figure 1. As one can see, the results of simple two orthogonal spheroids, simplified model and original model are in agreement. Therefore, we can ignore the exhaust flame fluctuations in HF frequencies.

![Figure 1](image-url)  

**Figure 1** RCS of civil aircraft (solid line), simple model (small dash line) and two orthogonal spheroids (long dash line).
The scattering results (multi static RCS) for simple cylinder with length 8m and radiuses 0.5m are shown in figure 2 (solid line). The RCS of a cylinder with exhaust (spheroid with radiuses 4m in behind) are shown with dashed line in figure 2.

Table 1: dimensions of original and simplified models

<table>
<thead>
<tr>
<th></th>
<th>Civil aircraft</th>
<th>Simplified model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main body</td>
<td>54 m</td>
<td>54 m</td>
</tr>
<tr>
<td>Main body diameter</td>
<td>5.6 m</td>
<td>2.5 m</td>
</tr>
<tr>
<td>Wing dimensions</td>
<td>$25 \times 5 \times 1.5$ m</td>
<td>$25 \times 0.8$ m</td>
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Figure 2 RCS of simple cylinder with and without exhaust

2. Exhaust simulations and phenomenology

Study of electromagnetic scattering from flying objects in the HF frequency depended on the identification and analyze of the produced particles from the engine of target. In HF frequencies, aircraft and missiles engine exhaust has noticeable RCS. The scattering process can be expanded by propagation and diffraction theories of electromagnetic waves in ionized regions (plasma medium). The density of electrons and ions has main rule in scattering process, and density depends on temperature and pressure distribution behind the exhaust. Atmosphere pressure also is important in the equation of state (EOS) of the exhaust products. Therefore, at various altitudes, the EOS of engine plume plasma is different. If the plasma frequency is greater than incoming RF frequency, then we can treat plasma medium as perfect electric conductor (PEC). The pattern of temperature and ion concentration of a typical exhaust of aircrafts and missiles are given in figures 3-6. As one can see, there is a direct relation between temperature and ion or electron density in engine products. By increasing the altitude of flight the dimensions of hot area are increased and this means that at medium altitudes the effect of engine plume RCS increases. If we use the measurement data of aircraft engine or simulated results for missile exhaust, we can find that the plasma frequency of engine plume plasma is greater than HF frequency. So if we use HF frequencies for detection of stealth targets the exhaust can increases the RCS of target. Figure 3 shows the measured data for temperature distribution behind the nozzle [4]. If we compare this figure with figures 3-5 we find that the electron density is about $10^9-10^{10}$ (ccm)$^{-1}$. If we use these results, we find that the plasma frequency is about $10^8$ Hz [2]. This value is greater than HF frequency (typically $10^7$) and plasma is in ultra-dense region. Therefore, we can treat the plasma as PEC. Figure 6 shows the measured data for an aircraft engine. The energy of particles i.e. electrons and ions, is proportional to $K_BT$, where $K_B$ is Boltzmann constant.

If we assume that temperature is about 2000 K, then the thermal energy is of order $10^{-1}$ eV. This is comparable with ionization energy of molecules and heavy ions and recombination of ions and electrons is thermodynamically favorable. So, we expect that the effect of engine plume for aircraft at low altitudes becomes small.

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Conclusions

We investigate the effect of exhaust flume effect in the RCS of missiles and aircrafts. Because of electron and ion distribution behind the engine, the plasma frequency of plume is greater than incoming RF frequency in HF region. So we can treat the plume as PEC and the RCS of it is added to RCS of main target. According to measurements and simulations, in medium altitudes the effect of exhaust is considerable.

References
