Science Journal of Physics
ISSN: 2276-6367
http://www.sjpub.org

Published By
Science Journal Publication
International Open Access Publisher

Research Article

New solutions of Lorentz transformation

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Accepted on February 15, 2016

Abstract: A simple thought experiment with light clock, which is known in connection with the theory of relativity and its modifications, led me to create the next paper. Paper relates to other mathematical solutions of Lorentz transformation. The original solution is includes in these new solutions, but is only valid in a specific case. Moreover these new solutions include the parameter alpha, thus they are also certain wave functions, which satisfy the conditions of invariance even if their calculations are performed by non-standard manner.

Keywords: Lorentz transformation, STR

Introduction

1. Lorentz Transformation

My counts begin by Lorentz transformation. This system of transformational equations was extract from postulates of STR (special theory of relativity) and general recognized conformation of this transformation is:

(1.1) x´ = k (x – vt) for system S

It is only rough estimate of transformation but it has three arguments:

1. It is linear in both system S and S´
2. It is simple
3. We can reduce it on relation x´= x – vt , how is valid in classic physic

Because relations must to have the same form in system S and in S´, adequate we will be to change a mark of v, so

x = k (x´ + vt´) for system S´

Assumption is: factor k is in both systems the same, because S and S´ diverge only in mark of speed.

I corrupt simplicity of transformation and presumable to write inverse conformation as:

(1.2) x = k´ (x´ + vt´) for system S'

When I cogitate that k = f(v) and k´= f(-v).

After substitution (1.1) to (1.2) is

x = kk´(x – vt) + k´vt´

So

(1.3) $t' = kt + \left(\frac{1-kk'}{k'v}\right) x$

From second postulate STR consequent that

(1.4) in system S x = ct

(1.5) and in system S´ x´= ct´

By induction (1.1) and (1.3) to equation (1.5) we'll get

$$k(x - vt) = ckt + \left(\frac{1 - kk'}{kv'}\right) cx$$

And resolution in respect of x is

$$x = \frac{ckt + vkt}{k - [(1 - kk')/k'v]c} = ct \left\langle \frac{k + kv/c}{k - [\langle 1 - kk' \rangle/k'v]c} \right\rangle = ct \left[\frac{1 + v/c}{1 - \left(\frac{1}{kk'} - 1\right) c/v} \right]$$

And if relation (1.4) x = ct hold then sub expression is 1, therefore

$$\left[\frac{1 + v/c}{1 - \left(\frac{1}{kk'} - 1\right) c/v} \right] = 1$$

How to Cite this Article: Tomáš Kafoněk, "New solutions of Lorentz transformation", Science Journal of Physics, Volume 2016, Article ID sjp-220, 7 Pages, 2016, doi: 10.7237/sjp/220

and

(1.6) $kk' = \frac{1}{1-\frac{v^2}{c^2}}$

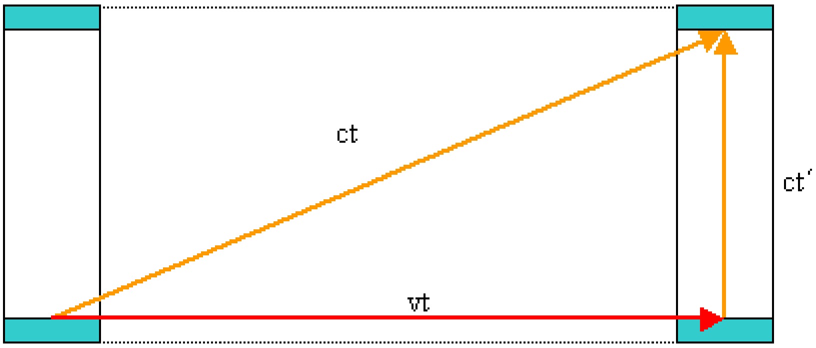
So if we assume k = k´ then

(1.7) $k = \frac{1}{\sqrt{(1-v^2/c^2)}}$

This is even function and really different marks of speed v have no effect.

2. Light Clock, Twins Paradox

For next it is inevitable to commemorate function of light clock. It is theoretical experiment, when we measure time on space ship sticking out by speed v from speculator. Time we measure by ray of light which we let to tick between two parallel mirrors. Pedigree of equations will be more simple if we'll see the picture:



In system S clock ticks once in time t, while in system S´ once in time t´. Triangle is square, then:

(2.1)

$$(ct)^2 = (vt)^2 + (ct')^2$$

$$(ct)^2 - (vt)^2 = (ct')^2$$

$$t^2(c^2 - v^2) = c^2t'^2$$

$$\frac{t^2}{t'^2} = c^2/(c^2 - v^2)$$

From here we count

(2.2)

$$\frac{t}{t'} = \frac{1}{\sqrt{(1 - v^2/c^2)}}$$

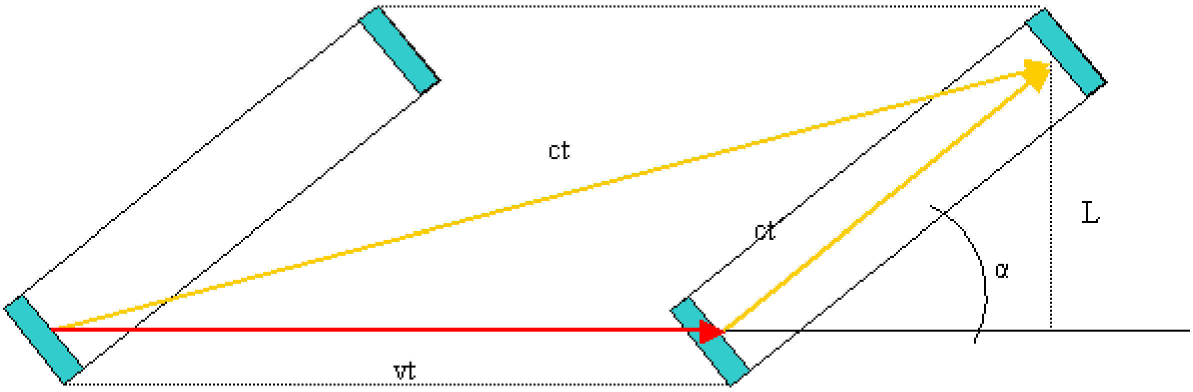
Accomplishment of this model accordant really with relation (1.7), which affirm Lorentz's theory. This carry one underlying problem if we'll be to philosophize over the relation. In literature is lettering as time dilation and the problem is twins paradox.

If I visualize I'm pilot of space ship, which speed approximate speed c, I don't age as quickly as my twice on earth. When I come back, I'm junior. First postulate of STR says space ship's speed from earth and earth's speed from space ship is the same. Relations on earth and on space ship are the same too. If we say one of twice is junior controvert ourselves assumptions.

Bare fact is every twice which come back from space path is as old as twice on earth. Science turn in another theory, on the other hand, time measuring of subatomic particles sets as demonstration of this theory.

3. Rotating Light Clock

Next time we'll get on theoretical experiment with light clock. We can ask what happened if this clock in space ship begins rotate. Picture helps our conception.



Then we can arrange equations for L by following method:

(3.1) $L^2 = (ct)^2 - (vt + ct'cos\alpha)^2$

(3.2) $L^2 = (ct')^2 - (ct'cos\alpha)^2$

By substitution (3.1) and (3.2) we get:

$(ct)^2 - (vt + ct'cos\alpha)^2 = (ct')^2 - (ct'cos\alpha)^2$

$(ct)^2 - (vt)^2 - 2vtct'cos\alpha - (ct'cos\alpha)^2 = (ct')^2 - (ct'cos\alpha)^2$

$(ct)^2 - (vt)^2 - 2vtct'cos\alpha = (ct')^2$

Because it's more simple we ascertain rate t´/t instead t/t´ :

$$1 - \frac{v^2}{c^2} - \left(\frac{2vcos\alpha}{c}\right)\left(\frac{t'}{t}\right) = \left(\frac{t'}{t}\right)^2$$

We can overwrite it on:

(3.3) $\left(\frac{t'}{t}\right)^2 + \left(\frac{2vcos\alpha}{c}\right)\left(\frac{t'}{t}\right) + \left(\frac{v^2}{c^2} - 1\right) = 0$

Equation's solution of type $x^2 + bx + c = 0$ is $x = \frac{\left[-b \pm \sqrt{b^2 - 4c}\right]}{2}$ so substitution into (3.3)

(3.4) $\frac{t'}{t} = -\frac{v}{c}cos\alpha \pm \sqrt{1 - \frac{v^2}{c^2}(1 - \cos^2\alpha)}$

This relation is more complicated then relation (2.2), but of course both bear on together, because for α=90˚ is cosα=0, so:

$$\frac{t'}{t} = \pm\sqrt{1 - \frac{v^2}{c^2}}$$

For next counts we choose only positive solution

$$\frac{t}{t'} = \frac{1}{\sqrt{(1 - \frac{v^2}{c^2})}}$$

If relation (1.7) and (2.2) are expression of the same deformation, therefore

$k = k' = \frac{t}{t'} = \frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$ for cosα=0

Then we can rewrite also (3.4) and we must respect mark of speed v

(3.5) $\frac{1}{k} = -\frac{v}{c}cos\alpha + \sqrt{1 - \frac{v^2}{c^2}(1 - \cos^2\alpha)}$

(3.6) $\frac{1}{k'} = \frac{v}{c}cos\alpha + \sqrt{1 - \frac{v^2}{c^2}(1 - \cos^2\alpha)}$

Now I show how these equations are in compliance with equations (1.1) and (1.2). It I can to demonstrate by multiplication (3.5) and (3.6)

$$\frac{1}{kk'} = \left[-\frac{v}{c}cos\alpha + \sqrt{1 - \frac{v^2}{c^2}(1 - \cos^2\alpha)}\right] * \left[\frac{v}{c}cos\alpha + \sqrt{1 - \frac{v^2}{c^2}(1 - \cos^2\alpha)}\right]$$

$$\frac{1}{kk'} = 1 - \frac{v^2}{c^2}(1 - \cos^2\alpha) - \frac{v^2}{c^2}\cos^2\alpha = 1 - \frac{v^2}{c^2}$$

$kk' = \frac{1}{1 - \frac{v^2}{c^2}}$ Accordant with relation (1.6)

So equations (3.5) and (3.6) create available resolution of Lorentz transformation.

4. Invariant under a Lorentz Transformation

It is assumed that the relation must be satisfied:

$x'^2 - c^2t'^2 = x^2 - c^2t^2$ (4.1)

But my calculations correspond to invariant relation:

$x'^2 - c^2t'^2 = c^2t^2 - x^2$ (4.2)

It is argued that applies equation (4.1), due to maintaining uniformity of the transformation for v=0. But I think that if v=0 this equation is also tied to relations $x'^2 = c^2t'^2, x^2 = c^2t^2$. Therefore then this is the equation:

$x'^2 - c^2t'^2 = c^2t^2 - x^2 = 0$ (4.3)

Additionally I counted with the following relationships:

$x' = A.x + B.t, t' = P.x + Q.t$ (4.4)

Substituting into equation (4.2) then we get:

$(A^2 - c^2P^2)x^2 + 2(AB - c^2PQ)x.t + (B^2 - c^2Q^2)t^2 = c^2t^2 - x^2$ (4.5)

Of which we compute the four basic equations:

$(A^2 - c^2P^2) = -1$ (4.6)

$(AB - c^2PQ) = 0$ (4.7)

$(B^2 - c^2Q^2) = c^2$ (4.8)

$A.v + B = 0$ (4.9)

Upon further sequence of calculations it is important to remember what i said relationship:

$\frac{t'}{t} = -\frac{v}{c}cos\alpha \pm \sqrt{1 - \frac{v^2}{c^2}(1 - \cos^2\alpha)}$ (3.4)

That I then generalized for other calculations I used only positive solution but the entire resolution relationship should take following form:

$\frac{1}{K} = -\frac{v}{c}cos\alpha \pm \sqrt{1 - \frac{v^2}{c^2}(1 - \cos^2\alpha)}$ (4.10)

So in relation to equations (4.4) we can write

$A_1 = \frac{1}{-\frac{v}{c}cos\alpha + \sqrt{1 - \frac{v^2}{c^2}(1 - \cos^2\alpha)}}$ (4.11)

$B_1 = \frac{-v}{-\frac{v}{c}cos\alpha + \sqrt{1 - \frac{v^2}{c^2}(1 - \cos^2\alpha)}}$ (4.12)

$A_2 = \frac{1}{-\frac{v}{c}cos\alpha - \sqrt{1 - \frac{v^2}{c^2}(1 - \cos^2\alpha)}}$ (4.13)

$B_2 = \frac{-v}{-\frac{v}{c}cos\alpha - \sqrt{1 - \frac{v^2}{c^2}(1 - \cos^2\alpha)}}$ (4.14)

If I consider that the solution for A1 and A2 / and B1 and B2 / apply simultaneously and expect a certain analogy, then gives equation (4.2) rewrite as:

$$(A_1x + B_1t)(A_2x + B_2t) - c^2(P_1x + Q_1t)(P_2x + Q_2t) = c^2t^2 - x^2$$

Then

$(A_1A_2 - c^2P_1P_2)x^2 + (A_1B_2 + A_2B_1 - c^2P_1Q_2 - c^2P_2Q_1)x.t + (B_1B_2 - c^2Q_1Q_2)t^2 = c^2t^2 - x^2$ (4.15)

$(A_1A_2 - c^2P_1P_2) = -1$ (4.16)

$(A_1B_2 + A_2B_1 - c^2P_1Q_2 - c^2P_2Q_1) = 0$ (4.17)

$(B_1B_2 - c^2Q_1Q_2) = c^2$ (4.18)

$A_1.v + B_1 = 0$ (4.19.1)

(Simultaneously)

$A_2.v + B_2 = 0$ (4.19.2)

These four relationships provide the basic requirements for the transformation and therefore I will discuss four points:

1) So if we start from equation (4.16) and substitute it into equation (4.11) and (4.13), then:

$P_1P_2 = \frac{A_1A_2+1}{c^2} = \frac{\frac{c^2}{v^2-c^2}+1}{c^2} = \frac{\frac{v^2}{v^2-c^2}}{c^2}$ (4.20)

2) For relations (4.11), (4.12), (4.13) and (4.14) hold:

$$A_1B_2 = A_2B_1 = -v.(\frac{c^2}{v^2 - c^2})$$

Substituting into (4.17) gives:

$$c^2(P_1Q_2 + P_2Q_1) = -2v.(\frac{c^2}{v^2 - c^2})$$

Then

$(P_1Q_2 + P_2Q_1) = -2v.(\frac{1}{v^2-c^2})$ (4.21)

3) From relation (4.18) can be deduced:

$Q_1Q_2 = \frac{c^2}{v^2-c^2}$ (4.22)

Relations (4.19.1) and (4.19.2) correspond to equations (4.11), (4.12), (4.13) and (4.14)

Relations (4.20), (4.21) and (4.22) correspond to equations:

$Q_1 = \frac{1}{-\frac{v}{c}cos\alpha + \sqrt{1 - \frac{v^2}{c^2}(1 - \cos^2\alpha)}}$ (4.23)

$Q_2 = \frac{1}{-\frac{v}{c}cos\alpha - \sqrt{1 - \frac{v^2}{c^2}(1 - \cos^2\alpha)}}$ (4.24)

$P_1 = -\frac{\frac{v}{c^2}}{-\frac{v}{c}cos\alpha + \sqrt{1 - \frac{v^2}{c^2}(1 - \cos^2\alpha)}}$ (4.25)

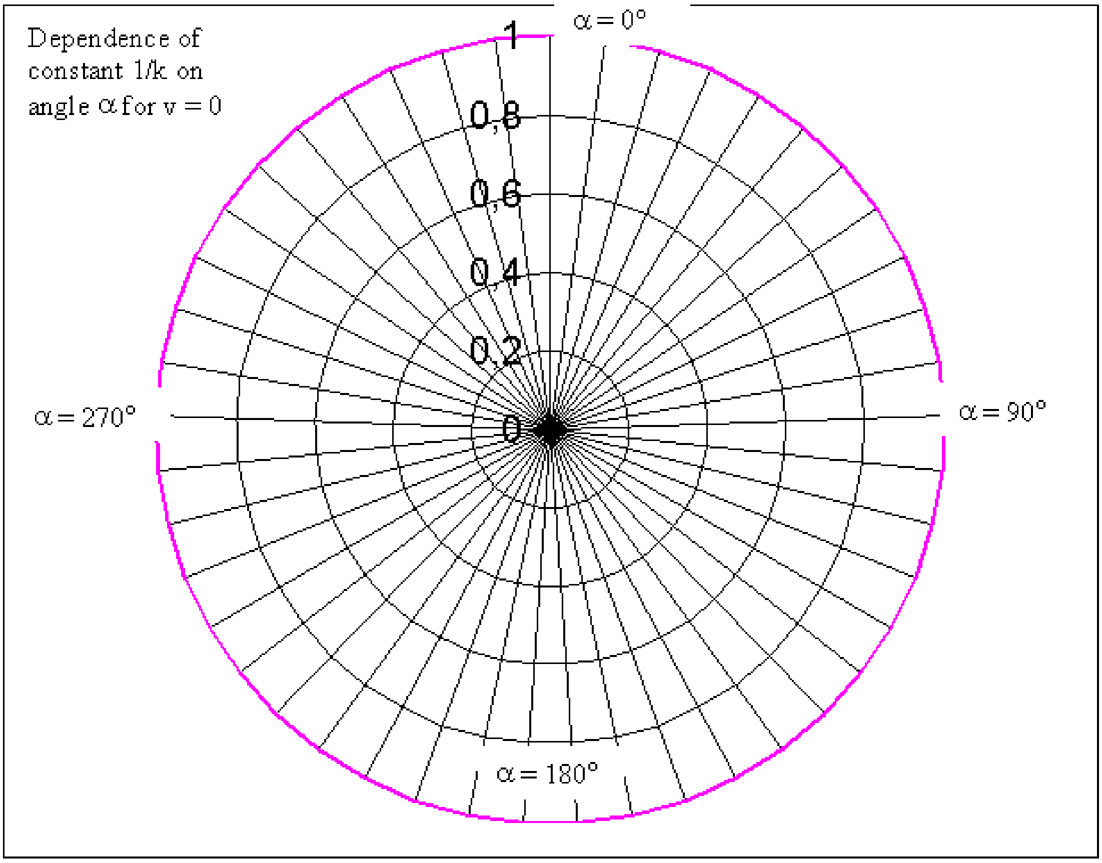
$$P_2 = -\frac{\frac{v}{c^2}}{-\frac{v}{c}cos\alpha - \sqrt{1-\frac{v^2}{c^2}(1-\cos^2\alpha)}} \qquad (4.26)$$

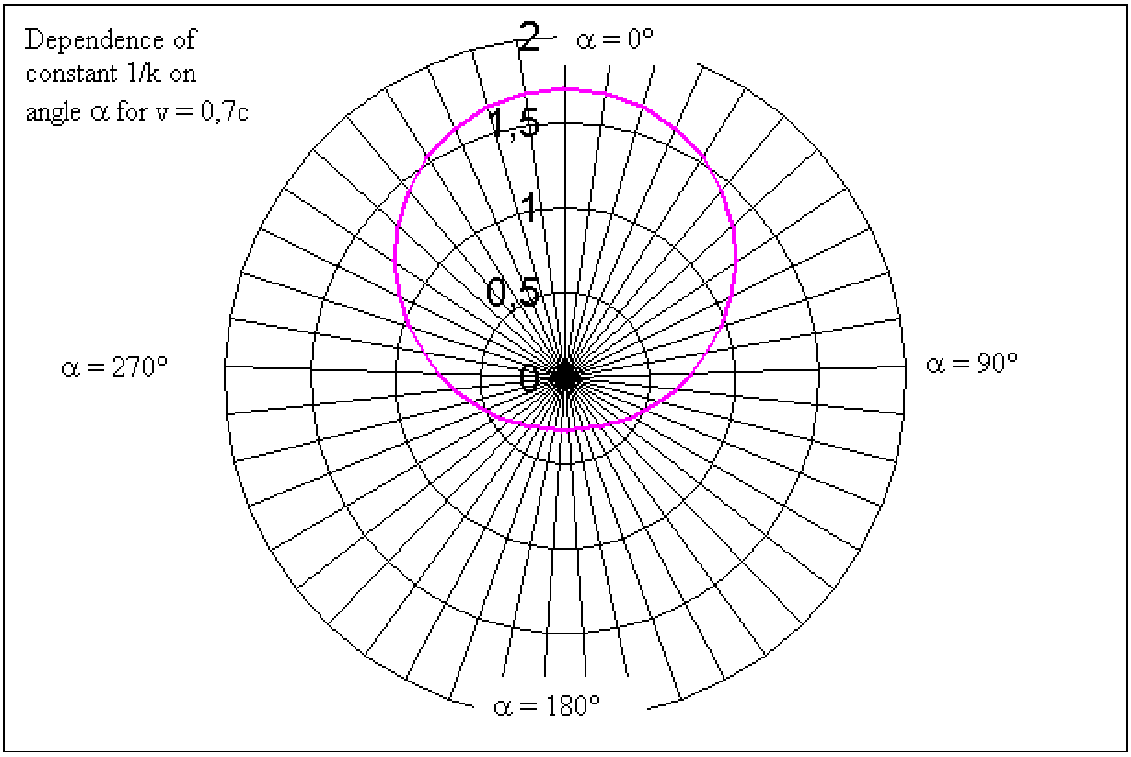
Solution of Lorentz transformation is as follows:

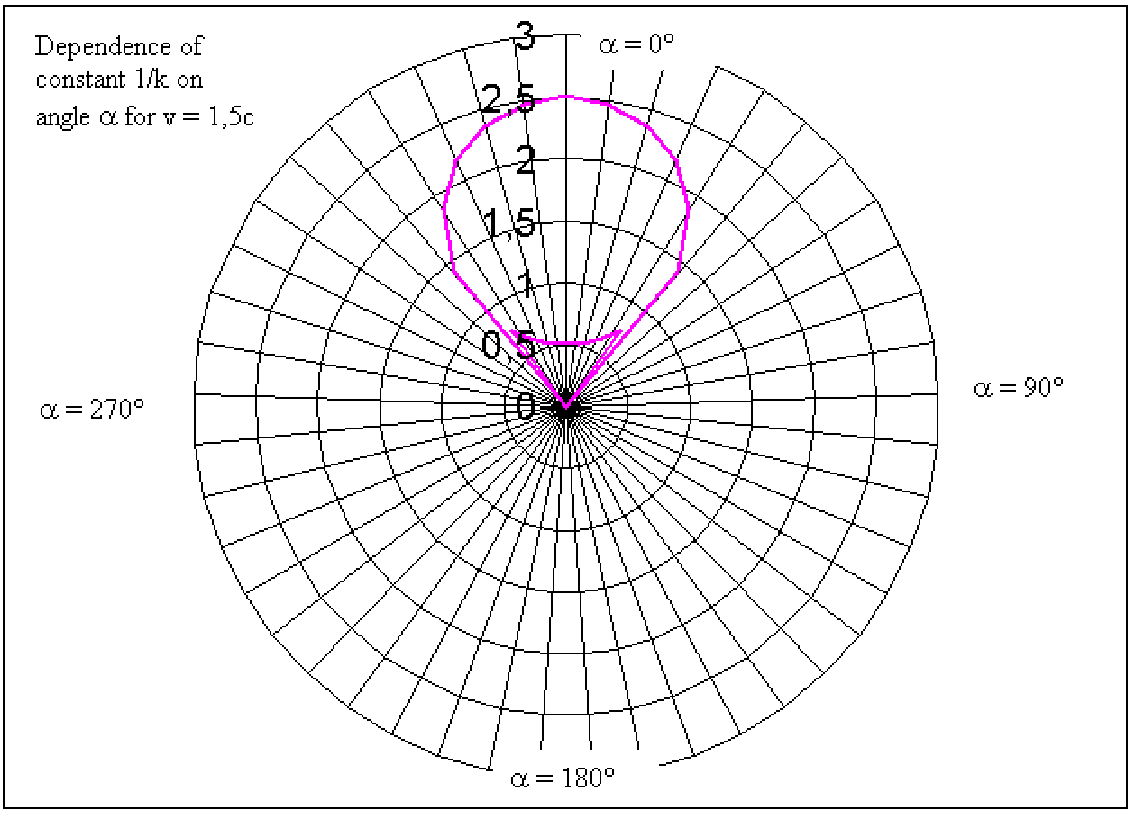
$$x' = \frac{1}{-\frac{v}{c}cos\alpha \pm \sqrt{1-\frac{v^2}{c^2}(1-\cos^2\alpha)}}.(x - v.t) \qquad (4.27)$$

$$t' = \frac{1}{-\frac{v}{c}cos\alpha \pm \sqrt{1-\frac{v^2}{c^2}(1-\cos^2\alpha)}}.(t - vx/c^2) \qquad (4.28)$$

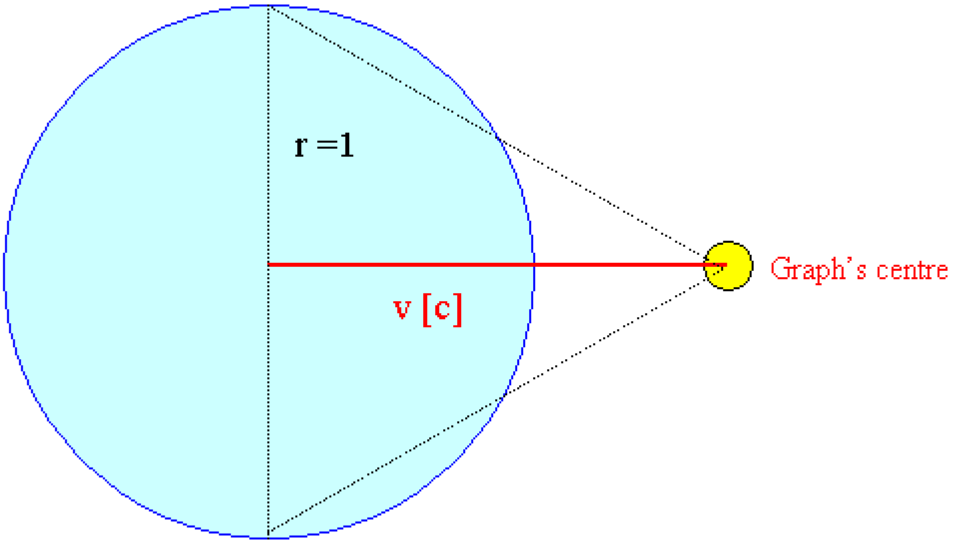
5. Graphs of function 1/k – relation (3.5)







These graphs have following character:



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How to Cite this Article: Tomáš Kafoněk, "New solutions of Lorentz transformation", Science Journal of Physics, Volume 2016, Article ID sjp-220, 7 Pages, 2016, doi: 10.7237/sjp/220