Characteristics of (lambda/10) 4-element Square Shape Stack Array Antenna for Terrestrial TV

E. P. Ogherowo, M. O. Alade

1 Department of Physics, University of Jos, P.M.B. 2084, Jos, Plateau State, Nigeria. E-mail: enohipius@yahoo.com

2 Department of Pure and Applied Physics, Ladoke Akintola University of Technology, P.M.B.4000, Ogbomoso, Oyo State, Nigeria. E-mail: aladey2008@yahoo.com

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ABSTRACT

In this paper, the design of (lambda/10) 4-Element Square Shape Stack Array Antenna is proposed for terrestrial television (TV) at 170 – 710MHz frequency band. Analytical equations were derived from the existing first principle electromagnetic models of a single loop antenna for the proposed antenna characteristics. MATLABR2007a was employed for the calculation of the proposed antenna characteristics. The results show that the gain of the antenna is between 7dBi and 16dBi as against the chosen design gain of 12dBi within the frequency band of consideration. Also, the antenna voltage standing wave ratio is between 1.1 and 2.4 within the frequency band of consideration. The antenna has demonstrated omnidirectional field patterns with -3dB beamwidth of 120°±5°. However, more than 89% of the incident power would be absorbed by the proposed antenna at the frequency range of 203.25 – 710MHz as illustrated by the results of the reflection coefficient and the return loss of the antenna, and in agreement with the earlier work by other authors. Therefore, this study has shown that the proposed antenna demonstrates high performance at the bandwidth of 203.25 – 710MHz for the terrestrial television network applications, (impedance bandwidth of ~4:1 is achieved).

KEYWORDS: Square Shape Stack Array, Antenna characteristics, Terrestrial television networks, Electromagnetic models, Numerical calculation.

INTRODUCTION

Due to the tremendous increase in the applications of wireless communications technology (Radio, TV and Mobile communication) in the last two decades, the design and analysis of wideband (WB) antenna structure that can maintain a desired high gain, provide efficiently tailored radiation patterns while allowing simplified impedance matching and assembly is required in order to increase the efficiency of the communication systems [1 – 4]. A major challenge facing the development of high performance modern antennas is how to simultaneously maximize the antenna design most basic requirements (parameters), namely bandwidth, gain, size, radiation pattern (field pattern), Voltage Standing Wave Ratio, reflection coefficient, return loss, and cost. This challenge is addressed in this study.

It is safe and economical to carry out the electromagnetic analysis of an antenna structure from the first principles as laid down by Maxwell, Marconis and others [4 – 7] in order to predict the performance of the device well in advance before embarking on practical implementation and experimental measurements to confirm the prediction. Most antenna engineers rely solely on the numerical electromagnetic code softwares (MMNEC, Vcal etc) for the antenna design and analysis. The result obtained from the NEC only analysis is sometime erroneous. Without adequate electromagnetic analysis of an antenna, the signal generated by the radio frequency (rf) system will not be transmitted efficiently and signal can not be faithfully detected by such antenna for further processing. However, for most antennas, their design parameters are so complex that closed form mathematical expressions are very difficult to formulate. Anyway, the easy way out is to integrate the analytical electromagnetic method with the numerical computation method.

Basic electromagnetic theory presentations have shown that at a distance from an antenna, the radiated spherical wave resembles a uniform plane wave. The electric field, \( \mathbf{E} \) and magnetic field, \( \mathbf{H} \) patterns characteristics of the plane waves radiated by the antenna can be obtained by solving the frequency domain Maxwell’s equations 1 and 2 [7] under the appropriate boundary conditions with the knowledge of the scalar potential, \( \Phi \) and vector potential, \( \mathbf{A} \) of the charge and current distributions respectively in the antenna.

\[
\mathbf{E} = -j\omega\mathbf{A} - \nabla \Phi, \quad \text{Vm}^{-1}
\]
\[
\vec{H} = \frac{1}{\mu} \nabla \times \vec{A} \quad \text{Am}^{-1}
\]  

(2)

Where, \( j = \sqrt{-1} \), \( \omega \) is the angular frequency and \( \mu \) is the permeability of the free space.

Hence, power radiated by the antenna is obtained from the Poynting vector expression given by [6 – 7]:

\[
P_{\text{rad}} = \frac{1}{2} |\vec{E} \times \vec{H}|^2 \quad \text{Watts}
\]  

(3)

The primary goal of this study is to design and analyze the performance of a simple, low-cost, and small size (\( \lambda / 10 \)) 4-element Square Shape Stack Array Antenna (at 170 – 710MHz band) that will provide wideband, high gain and omnidirectional field pattern properties such that there will be no need for rotational directional antenna types to detect terrestrial television networks at desired band. The integration of the analytical electromagnetic and the numerical computation methods are employed. The reason for the choice of Square Shape Stack Array Antenna is to combine the bandwidth compensation property of the loop antenna with the omnidirectional field pattern property of Stack Array Antenna [8], as an attempt to simultaneously maximize the antenna design requirements.

Therefore, this study presents the numerical electromagnetic predictions of the mutual impedances, radiation patterns, beamwidth, gain, VSWR, reflection coefficient, return loss and bandwidth properties of (\( \lambda / 10 \)) 4 Square Shape Stack Array Antenna at 170 – 710MHz band, based on the derived analytical expressions for the proposed antenna structure under consideration from the existing first principles electromagnetic equations available in the literatures [6 – 12].

1.0 Methods

2.1 Structural Design Analysis

Step 2: The Thickness of the conductor (element)

The thickness, \( t \) of the conductor element (Aluminium or Copper) employed is computed using the rule of thumb for thin linear antenna [7, 8, 12]:

\[
t \leq \frac{\lambda_{\text{mid}}}{100}
\]  

(9)

The value of the thickness chosen is 0.2cm.

Step 3: The Number of the conductor element

The chosen gain \( G \) of the antenna is 12dBi based on the experience of the gain of the local transmitting station antennas in Nigeria. Also, in other to achieve small size antenna medium low gain is chosen. The number (\( N \)) of the square loop element in the array is calculated using the

loop antenna structure is employed to provide wide bandwidth. Stack array configuration is formed to increase the gain of the antenna and still maintain omnidirectional property. This section presents the analysis of the step by step of the geometrical structure design of the proposed antenna in this study.

Step 1: The Length of the Square Loop

The middle (center) frequency of the band under consideration (170 – 710MHz) is 440MHz. From the first principle electromagnetic equation:

\[
\lambda = \frac{c}{f} \quad \text{m}
\]  

(4)

Where, \( \lambda \) is the wavelength, \( c \) is the speed of light in vacuum, and \( f \) is the resonant frequency. Therefore, the middle wavelength of the band under consideration is given as:

\[
\lambda_{\text{mid}} = \frac{3 \times 10^8 \text{ms}^{-1}}{440 \times 10^6 \text{Hz}} = 68.18 \text{cm},
\]  

(5)

For small size antenna, chosen length, \( L \) of the square loop is given as [7 – 8]:

\[
L \leq \frac{\lambda_{\text{mid}}}{10} = 6.8 \text{cm},
\]  

(6)

The actual chosen length, \( L \) of the square loop antenna in this study is 4.5cm.

The chosen length, \( L \) of the square loop antenna is in agreement with the condition for small loop antenna that [8, 12]:

\[
C_i \leq \frac{1}{3},
\]  

(7)

\[
C_i = \frac{\text{Loop Circumference}}{\lambda},
\]  

(8)

The value of the thickness chosen is 0.2cm.

Although, a single linear dipole antenna is omnidirectional antenna, its gain and bandwidth are very low. The folded dipole (loop) antenna on the other hands has bandwidth compensation and omnidirectional properties; it has the same gain as the linear dipole [8]. Therefore, in this study
formula \([7 - 8]\):
\[
G = N,
\]
(10)

Where, \( \pi = 3.142 \). The computed value of \( \bar{N} = 3.82 \), therefore the chosen \( N = 4 \).

**Step 4: The Spacing between the elements**

The optimized uniform Spacing between the elements of the array is computed using the expression \([8 - 12]\):
\[
s \leq \frac{s_{mini}}{20},
\]
(11)

Therefore, in order to achieve small boom length for the proposed antenna, \( s = 1 \text{cm} \) is chosen.

**Step 5: Impedance Matching and the Structural Configuration**

Normally, the characteristic impedance, \( Z \) of a loop antenna is 288ohms. In order to match the antenna with the 75ohms (R) coaxial transmission line, a U-shape \( \frac{\lambda}{4} \) coaxial cable matching techniques (coaxial cable balun) is suggested. This provides VSWR ~1.04. Figure 1 shows the configuration and dimension of the proposed antenna. The boom length \( B = L + h_3 \).

![Diagram showing the configuration and dimensions of the proposed antenna](image)

**Figure 1:** Showing the Configuration and Dimensions of the Proposed Antenna.

### 2.2 Calculation of the Antenna Characteristics

The basic equations employed to determine the Characteristics of the proposed antenna are described as follows:

**The Antenna Mutual Impedance**

The mutual impedance \( Z_{nm} \) of the array antenna is given by the expression \([7, 12]\):
\[
Z_{nm} = R_{nm} + jX_{nm},
\]
(12)

The reactive imaginary part, \( X_{nm} \) of the antenna impedance results from the electric and magnetic fields (close to the antenna) returning energy to the antenna during every cycle. The resistive part, \( R_{nm} \) consists of the ohmic losses (heat losses), \( R_L \) and radiation resistance, \( R_r \). The knowledge of antenna impedance would provide how much of the energy is absorbed by the antenna.

From the existing analytical expression \([8]\), the proposed antenna ohmic losses which are due to heating of the antenna by rf current passing through is formulated and given as:
\[
R_L = \frac{L}{\pi} \left[ \frac{2\omega \mu_0}{\sigma} \right],
\]
(13)

Where, \( \omega \) is the angular frequency \( (\omega = 2f) \), \( \mu_0 \) is the permeability in vacuum and \( \sigma \) is the conductivity of the conductor element.

The knowledge of mutual impedance \( Z_{nm} \) between two elements \((n \text{ and } m)\) dipole stack array antenna presented by...
Kraus (1988) [7] is applied to the present situation to derive the following equations:

\[ R_{o} = -15\cos\beta \left[ -2\alpha(\beta + \alpha)(\beta - L) + \alpha(\beta + \alpha) - \ln \left( \frac{h_{1}^{2} - L^{2}}{h_{2}^{2}} \right) \right] + 15\sin\beta \left[ 2\alpha(\beta - L) - \alpha(\beta + \alpha) \right], \]

\[ R_{o} = -15\cos\beta \left[ -2\alpha(\beta + \alpha)(\beta - L) + \alpha(\beta + \alpha) - \ln \left( \frac{h_{1}^{2} - L^{2}}{h_{2}^{2}} \right) \right] + 15\sin\beta \left[ 2\alpha(\beta - L) - \alpha(\beta + \alpha) \right], \]

\[ X_{o} = -15\cos\beta \left[ 2\alpha(\beta - L) - \alpha(\beta + \alpha) \right] + 15\sin\beta \left[ 2\alpha(\beta - L) - \alpha(\beta + \alpha) \right], \]

\[ X_{o} = -15\cos\beta \left[ 2\alpha(\beta - L) - \alpha(\beta + \alpha) \right] + 15\sin\beta \left[ 2\alpha(\beta - L) - \alpha(\beta + \alpha) \right], \]

\[ X_{o} = -15\cos\beta \left[ 2\alpha(\beta - L) - \alpha(\beta + \alpha) \right] + 15\sin\beta \left[ 2\alpha(\beta - L) - \alpha(\beta + \alpha) \right], \]

\[ \beta = \frac{2\pi}{\lambda}, \ h_{1} = L + s, \ h_{2} = 2h_{1}, \ h_{3} = 3h_{1} \]

Where, \( Z_{m} \) is the self impedance or input impedance.

**The Antenna Radiation Field Pattern**

The radiation field pattern factor of a single loop antenna of any size assuming uniform charges, current and voltage distributions is given as [7 - 9]:

\[ E_{r}(\theta) = J_{1}(C_{a}\sin\theta), \quad (20) \]

Where, \( C_{a} \) is the circumference of the square loop (in this case) in wavelength, \( \theta \) is the azimuthal angle in the horizontal plane about the vertical plane and \( J_{1} \) is the first-order Bessel function.

Since \( L \) is small (\( L \ll \lambda \)), as the power of \( J_{1} \) increases, the corresponding finite term in the series become insignificant and therefore \( E_{r}(\theta) \) can be approximated to:

\[ E_{r}(\theta) = \frac{2L}{\lambda} \sin\theta - \frac{L}{\lambda} \sin^{3}\theta + 2\frac{L}{\lambda} \sin^{5}\theta, \quad (21) \]

By mathematical arrangement and application of De-Moivre's theorem [13], \( E_{o}(0) \) for the proposed antenna (the single element) becomes:

\[ E_{o}(\theta) = a_{1} \sin\theta + a_{2} \sin3\theta + a_{3} \sin5\theta, \quad (22) \]

Where,

\[ a_{1} = \frac{2L}{\lambda} - \frac{L}{\lambda} \sin^{3}\theta + 2\frac{L}{\lambda} \sin^{5}\theta, \]

\[ a_{2} = \frac{L}{\lambda} - 0.8125 \frac{L}{\lambda} \sin^{5}\theta, \quad (23) \]

Therefore, the relative field pattern of the 4-element square shape stack array antenna in this study, according to the principle of pattern multiplication, can be expressed as:

\[ E(\theta) = 4E_{o}(\theta), \quad (24) \]

**The Gain in Field Intensity of the Proposed Antenna**

The gain in field intensity, \( |G_{r}(\theta)| \) of an array antenna can be defined as the ratio of the relative field intensity, \( |E(\theta)| \) due to the array to the relative field intensity, \( |E_{o}(\theta)| \) due to a single element of the array [7, 8, 12].

The relative field intensity due to the single square element of the proposed antenna can be written as:

\[ |E_{o}(\theta)| = kV_{o}(a_{1} \sin\theta + a_{2} \sin3\theta + a_{3} \sin5\theta), \quad (25) \]

Where, \( V_{o} \) is the voltage distribution due to a single element and \( k \) is a constant with the unit \( m^{-1} \).

By the principle of pattern multiplication:

\[ |E_{r}(\theta)| = |E_{r}(\theta)| = |E_{r}(\theta)| = |E_{r}(\theta)| = |E_{r}(\theta)|, \quad (26) \]

Where,

\[ |E_{r}(\theta)| \] is the field intensity due to the conductor element 1;\n
\[ |E_{r}(\theta)| \] is the field intensity due to the conductor element 2;\n
\[ |E_{r}(\theta)| \] is the field intensity due to the conductor element 3;\n
\[ |E_{r}(\theta)| \] is the field intensity due to the conductor element 4;

The total field intensity due to the proposed antenna is given as:

\[ |E(\theta)| = 4|E_{o}(\theta)| = 4|E_{o}(\theta)|, \quad (27) \]

\[ |E(\theta)| = 4kV_{o}(a_{1} \sin\theta + a_{2} \sin3\theta + a_{3} \sin5\theta), \quad (28) \]

Note, \( V_{o} = V_{1} = V_{2} = V_{3} = V_{4} = V_{l} \).

The real total power, \( P \), input to the antenna, since \( R_{l} \) in Equation 12 is very small (negligible), is purely radiation resistance and can be written as:

\[ P = \frac{V^{2}}{R}, \quad \text{Watts} \quad (29) \]
The power input to the conductor element 1 is given as:

\[ P_1 = \frac{V_1^2}{(R_{11} + R_{12} + R_{13} + R_{14})}. \]  

(30)

The power input to the conductor element 2 is given as:

\[ P_2 = \frac{V_2^2}{(R_{22} + R_{21} + R_{23} + R_{24})}. \]  

(31)

The power input to the conductor element 3 is given as:

\[ P_3 = \frac{V_3^2}{(R_{33} + R_{31} + R_{32} + R_{34})}. \]  

(32)

The power input to the conductor element 4 is given as:

\[ P_4 = \frac{V_4^2}{(R_{44} + R_{41} + R_{42} + R_{43})}. \]  

(33)

Since the shape, size and spacing of the proposed array are uniform geometrically:

\[ R_{oo} = R_{11} = R_{22} = R_{33} = R_{44}; R_{12} = R_{21}; R_{13} = R_{31}; R_{14} = R_{41}; R_{23} = R_{32}; R_{24} = R_{42}; R_{34} = R_{43}; R_{12} = R_{23} = R_{34}; R_{13} = R_{24}. \]

Then, total power P is:

\[ P = P_1 + P_2 + P_3 + P_4 = \frac{2M^2}{R_{oo}^2 + R_{oo}^2 + R_{oo}^2 + R_{oo}^2} \left( \frac{2N^2}{R_{oo}^2 + R_{oo}^2 + R_{oo}^2 + R_{oo}^2} \right). \]

(34)

\[ \therefore V_i = \frac{P(R_{11} + R_{12} + R_{13} + R_{14})(R_{12} + R_{13} + R_{14} + R_{11})}{2(2R_{11} + 3R_{12} + 2R_{13} + R_{14})}. \]

(35)

Assuming the same total power is input to a single conductor element of the array, then:

\[ P = \frac{V^2}{R_{oo}}, \]

(36)

\[ V_o = \sqrt{PR_{oo}}, \]

(37)

\[ |E_i(\theta)| = \sqrt{PR_{oo}} \left( \frac{a_i \sin \theta + a_j \sin 3\theta}{2a_i + 3a_j + 2R_{12} + R_{13}} \right), \]

(38)

\[ |E_x(\theta)| = k \sqrt{PR_{oo}} \left( a_i \sin \theta + a_j \sin 3\theta \right). \]

(39)

By the definition of \( G_f(\theta) \), the null of the field occurs only when the numerator of \( G_f(\theta) \) is zero. Therefore, the Gain in Field Intensity of the Proposed Antenna can be expressed as:

\[ G_f(\theta) = \frac{|E_i(\theta)|}{|E_x(\theta)|^2} \left( \frac{R_{11} + R_{12} + R_{13} + R_{14}}{R_{12} + R_{13} + R_{14} + R_{11}} \right) \left( a_i \sin \theta + a_j \sin 3\theta \right), \]

(40)

The Voltage Standing Wave Ratio, Reflection Coefficient and Return Loss of the Antenna

The analytical equations are derived for the Voltage Standing Wave Ratio (VSWR), Reflection Coefficient (ρ) and Return Loss (\( \alpha \)) properties of the proposed Antenna and present as follows [8, 14, 15]:

\[ \text{VSWR} = 1.4118^2, \]

(41)

\[ \rho = \frac{1.4118-1}{1.4118+1}, \]

(42)

\[ \alpha = -20 \log_{10} \frac{1.4118^2-1}{1.4118^2+1}. \]

(43)

2.0 Results and Discussion

In this paper, numerical calculation of the performance characteristics of the proposed WB (\( \lambda/10 \)) 4 Square Shape Stack Array Antenna for Terrestrial TV at 170 – 710MHz is presented. The proposed antenna is designed and the structural configuration is shown in the Figure 1. The analytical equations were derived from the existing electromagnetic models for the proposed antenna characteristics, namely: the mutual impedances, radiation field patterns, gain in field intensity, VSWR, reflection coefficient, and return loss. Fourteen transmitting stations; 170.0, 175.25, 180.25, 203.25, 210.35, 217.25, 224.25, 440.0, 528.0, 583.15, 590.0, 606.25, 702.25 and 710.0MHz in the Western Region of Nigeria within the desired band were considered. The results of the numerical analysis of the proposed antenna characteristics using MATLABR2007a show that the antenna possesses excellent performance over the resonant frequencies considered.

Figure 2 shows the plot of the computed mutual impedance characteristic of the antenna versus the frequency. Both the resistive and reactive components of the antenna impedance are negative over the frequency band under consideration. The result shows that all the rf energy is absorbed by the antenna, and besides the antenna ohmic loss is insignificant as shown by the Equation 12.

Figures 3–16 show the plot of the computed radiation field patterns characteristic of the antenna at the horizontal plane (azimuthal angle, \( 0^\circ \leq \theta \leq 360^\circ \)) for the resonant frequencies under consideration. The result shows that the radiation field pattern characteristic of the antenna is omnidirectional. The beamwidth obtained is 120°±5° for all the resonant frequencies under consideration.

Figure 17 is the plot of the computed VSWR versus the frequency of the proposed antenna. VSWR varies between 1.1 and 2.4 within the desired band. Figure 18 is the plot of the reflection coefficient versus the frequency of the antenna. The reflection coefficient is less than –0.3 (the standard figure of merit [8]) at the resonant frequency between 203.25MHz and 710MHz. Figure 19 is the plot of the return loss versus the frequency of the antenna. The return loss is greater than 9.53dB (the standard figure of merit [8]) at the resonant frequency between 203.25MHz and 710MHz. This implies that the realized impedance bandwidth for the proposed antenna is 203.25 – 710MHz (–4:1). The computed antenna gain versus frequency is
shown in the Figure 19. The computed gain varies between 7dBi and 16dBi over the band of consideration as against the chosen gain of 12dBi.
Figure 4: Showing the antenna computed radiation pattern in the horizontal plane at 175.25MHz.

Figure 5: Showing the antenna computed radiation pattern in the horizontal plane at 180.25MHz.
Figure 6: Showing the antenna computed radiation pattern in the horizontal plane at 203.25MHz.

Figure 7: Showing the antenna computed radiation pattern in the horizontal plane at 210.35MHz.
Figure 8: Showing the antenna computed radiation pattern in the horizontal plane at 217.25MHz.

Figure 9: Showing the antenna computed radiation pattern in the horizontal plane at 224.25MHz.
Figure 10: Showing the antenna computed radiation pattern in the horizontal plane at 440MHz.

Figure 11: Showing the antenna computed radiation pattern in the horizontal plane at 528MHz.
Figure 12: Showing the antenna computed radiation pattern in the horizontal plane at 583.15MHz.

Figure 13: Showing the antenna computed radiation pattern in the horizontal plane at 590MHz.
Figure 14: Showing the antenna computed radiation pattern in the horizontal plane at 606.25MHz.

Figure 15: Showing the antenna computed radiation pattern in the horizontal plane at 702.25MHz.
Figure 16: Showing the antenna computed radiation pattern in the horizontal plane at 710MHz.

Figure 17: Showing the antenna computed VSWR versus Frequency.
Figure 18: Showing the antenna computed Reflection Coefficient versus Frequency.

Figure 19: Showing the antenna computed Return Loss versus Frequency.
Conclusions

In this paper, calculation of the proposed (λ/10) 4-Element Square Shape Stack Array Antenna for terrestrial TV at 170 – 710MHz frequency band is presented. Analytical equations were derived from the existing first principle electromagnetic models of a single loop antenna for the proposed antenna characteristics: the mutual impedances, radiation field patterns, gain in field intensity, VSWR, reflection coefficient, and return loss. MATLABR2007a was employed for the simulations of the proposed antenna characteristics. The calculated gain and VSWR of the proposed antenna fluctuate between 7dBi and 16dBi; and 1.1 and 2.4 respectively at 170 – 710MHz frequency band of consideration. The antenna demonstrates omidirectional field patterns with the beamwidth of 120°±5° at all the resonant frequencies within frequency band of consideration. However, more than 89% of the incident power is absorbed by the proposed antenna at the frequency range of 203.25 – 710MHz as illustrated by the results of the reflection coefficient and the return loss of the antenna. It shows that the impedance bandwidth of ~4:1 is realized by the proposed antenna. Eleven stations out of the fourteen transmitting TV stations considered, within the frequency band of consideration, will be faithfully detected by the antenna, which show better performance than the existing antenna that can not received more than two TV signals effectively without rotating the antenna. Therefore, the proposed antenna has demonstrated excellent performance in comparison with the existing antennas for terrestrial TV signals receptions at the same frequency band.

References


