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Some Properties of the Function Corresponding to Analysis of Time Series

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Abstract: We consider the problem of studying the properties of the function $\Phi^{(T)}(x,y)$ which corresponding to analysis of time series by using data window in two different cases; the first in discrete time series and the other in continuous time series. Some examples for the function $\Phi^{(T)}(x,y)$ and data window are discussed.

Keywords: Continuous time series, Data window, Dirac delta function.

1. Introduction

The properties of the function $\Phi^{(T)}(x,y)$ which corresponding to analysis of time series are considered. Also, the properties of the data window function $h^{(T)}(t)$which is bounded and equal zero outside the interval [0,T) are studied in many authors, as e.g. ([1] – [4]), [6]. In this paper we study the properties of the function $\Phi^{(T)}(x,y)$ in the discrete and continuous time processes. In Section 2 we introduce the notation and some recent results which will be used later. In Section 2 we introduce the notations and some recent results which will be used later. In Section 3 the properties of the function $\Phi^{(T)}(x,y)$ in the discrete case are given. Section 4 containes the properties of the function $\Phi^{(T)}(x,y)$ in the continuous case. Examples in the continuous case of data window function $h^{(T)}(t)$ which is bounded and equal zero outside the interval [0,T) and the function $\Phi^{(T)}(x,y)$ which is symmetric are given in Section 5.

2. Preliminaries

Let $h^{(T)}(t)_{(t=0,\pm1,\ldots)}$ be a function equal to zero outside the interval [0,T). The function $h^{(T)}(t)$ is called a data window, which is bounded. The function $\Phi^{(T)}(x,y)$ in discrete time series is defined by

$$\Phi^{(T)}(x,y) = (2\pi T)^{-1}\varphi^{(T)}(X)\varphi^{(T)}(y) \qquad \textbf{(2.1)}$$

where

$$\varphi^{(T)}(X) = \sum_{t=0}^{T-1} h^{(T)}(t)\exp(-ixt) \qquad \textbf{(2.2)}$$

and the bar denotes complex conjugate.

Also, the function $\Phi^{(T)}(x,y)$ for strictly stationary continuous time processes X(t) with observations $X(t),\ 0 \le t \le T$ is defined by

$$\Phi^{(T)}(x,y) = \frac{1}{2\pi\int_0^T \left(h^{(T)}(t)\right)^2 dt}\varphi^{(T)}(X)\varphi^{(T)}(y) \qquad \textbf{(2.3)}$$

Where

$$\varphi^{(T)}(x) = \int_0^T h^{(T)}(t)\exp(-i\,x\,t)\,dt, \qquad \textbf{(2.4)}$$

And $h^{(T)}(t)$, $(-\infty < t < \infty)$ is equal zero outside the interval $[0,T)$.

3. Properties of the function $\Phi^{(T)}(x,y)$ of the discrete time series

Now, the properties of the function $\Phi^{(T)}(x,y)$ in the discrete time series shall be studied by the following theorem:

Theorem 3.1 Suppose $h^{(T)}(t)$, $(t = 0, \pm 1, \ldots)$ is bounded and equal zero outside the interval $[0,T)$. Then for $(-\pi \le \lambda \le \pi)$,

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(1) $\int_{-\pi}^{\pi} \Phi^{(T)}(\upsilon-\lambda_1, \upsilon-\lambda_2)\, d\upsilon = \frac{1}{T} \sum_{t=0}^{T-1} \left(h^{(T)}(t)\right)^2 \exp\{-i(\lambda_1-\lambda_2)t\}.$ **(3.1)**

(2) $\int_{-\pi}^{\pi} \Phi^{(T)}(\upsilon-\lambda_1, \upsilon-\lambda_2)\, d\upsilon = 1,$ **(3.2)**

when $\lambda_1 = \lambda_2 = \lambda$.

(3) $\int_{-\pi}^{\pi} \Phi^{(T)}(\upsilon-\lambda_1, \upsilon-\lambda_2)\, d\upsilon = \frac{1}{T} \frac{\sin(\lambda_1-\lambda_2)T/2}{\sin(\lambda_1-\lambda_2)/2} \exp\{-i(\lambda_1-\lambda_2)(T-1)/2\}$

When $h^{(T)}(t) = 1$ and $\lambda_1 \neq \lambda_2$. **(3.3)**

(4) $\int_{-\pi}^{\pi} \Phi^{(T)}(\upsilon-\lambda_1, \upsilon-\lambda_2)\, d\upsilon = \begin{cases} 1 & s_1-s_2 = 0, \pm T, \pm 2T, \ldots \\ 0 & \text{otherwise}, \end{cases}$ **(3.4)**

When $h^{(T)}(t) = 1$, $\lambda_{s_i} = \frac{2\pi s_i}{T}$, i = 1, 2 and $\lambda_{s_1} \neq \lambda_{s_2}$.

The proof of this theorem is omitted.

4. Properties of the function $\Phi^{(T)}(x,y)$ of the continuous time series

Theorem 4.1 below studied the properties of the function $\Phi^{(T)}(x,y)$ in the continuous time series.

Theorem 4.1. Suppose $h^{(T)}(t)$, ($-\infty < t < \infty$) is a data window function which is bounded and equal zero outside the interval $[0,T)$. Then for all $\lambda_1, \lambda_2 \in R$,

(1) $\int_{-\infty}^{\infty} \Phi^{(T)}(\upsilon-\lambda_1, \upsilon-\lambda_2)\, d\upsilon = \frac{1}{\int_0^T \left(h^{(T)}(t)\right)^2 dt} \int_0^T \left(h^{(T)}(t)\right)^2 \exp\{-i(\lambda_1-\lambda_2)t\}\, dt$ **(4.1)**

(2) $\int_{-\infty}^{\infty} \Phi^{(T)}(\upsilon-\lambda_1, \upsilon-\lambda_2)\, d\upsilon = 1,$ **(4.2)**

when $\lambda_1 = \lambda_2 = \lambda$ **.**

(3) $\int_{-\infty}^{\infty} \Phi^{(T)}(\upsilon-\lambda_1, \upsilon-\lambda_2)\, d\upsilon = \frac{1}{T} \frac{\sin(\lambda_1-\lambda_2)T/2}{(\lambda_1-\lambda_2)/2} \exp\{-i(\lambda_1-\lambda_2)T/2\},$ **(4.3)**

when $h^{(T)}(t)$ **= 1 and** $\lambda_1 \neq \lambda_2$**.**

(4) $$\int_{-\infty}^{\infty} \Phi^{(T)}\left(\upsilon-\lambda_1,\upsilon-\lambda_2\right)d\upsilon = \begin{cases} 1 & s_1 - s_2 = 0, \pm T, \pm 2T, \ldots \\ 0 & \text{otherwise}, \end{cases} \quad \textbf{(4.4)}$$

when $h^{(T)}(t)$ **= 1,** $\lambda_{s_i} = \frac{2\pi s_i}{T}$ **, i = 1, 2 and** $\lambda_{s_1} \neq \lambda_{s_2}$.

Proof. Using the formula (2.3) and then using the given conditions, we get

$$\int_{-\infty}^{\infty} \Phi^{(T)}\left(\upsilon-\lambda_1,\upsilon-\lambda_2\right)d\upsilon = \frac{1}{\int_0^T \left(h^{(T)}(t)\right)^2 dt} \int_0^T \delta(t_1 - t_2) \times$$

$$\times \left[\int_0^T \left(h^{(T)}(t)\right)^2 \exp\{-i(\lambda_1 - \lambda_2)t\} dt_2 \right] dt_1,$$

where $\delta(t_1 - t_2)$ is the dirac delta function (for the definition see [5]). Then the property (4.1) follows. The property (4.2) follow directly from property (4.1) by setting $h^{(T)}(t)$ = 1 and $\lambda_1 = \lambda_2$. Putting $\lambda_1 \neq \lambda_2$ in property (4.1), we have

$$\int_{-\infty}^{\infty} \Phi^{(T)}\left(\upsilon-\lambda_1,\upsilon-\lambda_2\right)d\upsilon = \frac{1}{T}\int_0^T \exp\{-i(\lambda_1 - \lambda_2)t\} dt ,$$

and using the formula which says that

$$\int_0^T \exp\{-i(\lambda_1 - \lambda_2)t\} dt = \frac{\sin(\lambda_1 - \lambda_2)T/2}{(\lambda_1 - \lambda_2)/2} \exp\{-i(\lambda_1 - \lambda_2)T/2\}, \quad \textbf{(4.5)}$$

Then property (4.3) is obtained. Finally property (4.4) follows directly from property (4.3) by using the given conditions and the formula

$$\int_0^T \exp\{-i\,2\pi s\,t/T\} dt = \begin{cases} T & s = 0, \pm T, \pm 2T, \ldots \\ 0 & \text{otherwise}, \end{cases} \quad \textbf{(4.6)}$$

Which completes the proof of the theorem.

5. Examples

In this section we shall give tow examples show the properties of the data window function $h^{(T)}(t)$ and the function$\Phi^{(T)}(\lambda)$, (- ∞ < λ < ∞).

Our first example is selecting a data window function $h^{(T)}(t)$ of the form

$$h^{(T)}(t) = \frac{1}{2}\left(1 + \cos\frac{\pi t}{T}\right), \textbf{ for all } t \in \left(-T, T\right), \quad \textbf{(5.1)}$$

Which bounded and equal zero outside the interval $\left[0, T\right)$. By using numerical simulation methods, figures 1 and 2 show the properties of the data window function (5.1) for T = 4 and T = 8 respectively.

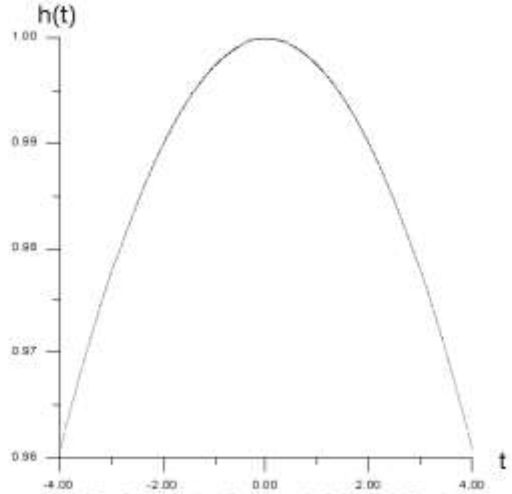

Fig.1. The Data Window Function of (2.2.12) with T=4

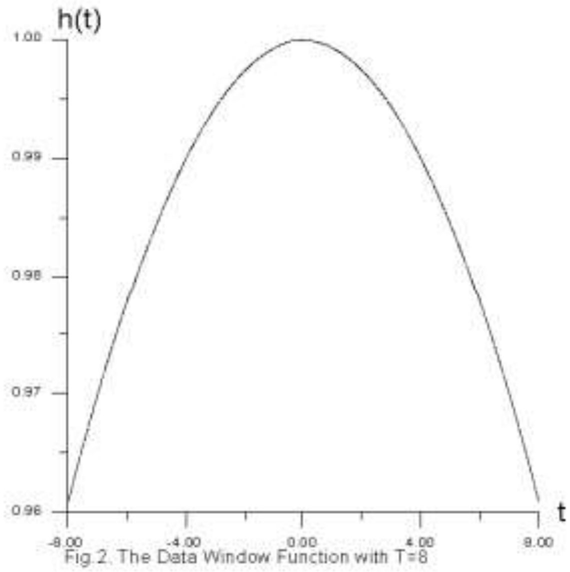

Fig.2. The Data Window Function with T=8

Generally speaking, for a data window function $h^{(T)}(t)$, its boundedness and its value vanish outside the interval $[0, T)$ is hold from the previous figures.

Our next example is a kernel $\Phi^{(T)}(\lambda)$, $(-\infty < \lambda < \infty)$,

$$\Phi^{(T)}(\lambda) = \frac{1}{2\pi \int_0^T \left(h^{(T)}(t)\right)^2 dt} \varphi^{(T)}(\lambda)\,\overline{\varphi^{(T)}(\lambda)}, \tag{5.2}$$

Where $\Phi^{(T)}(\lambda)$ is defined by formula (2.4) with data window function $h^{(T)}(t)$ given by (5.1). Figures 3, 4 and 5 show the properties of the function$\Phi^{(T)}(\lambda)$, which is given by (5.2) that is symmetric for T = 50, T = 10 and T = 20 respectively.

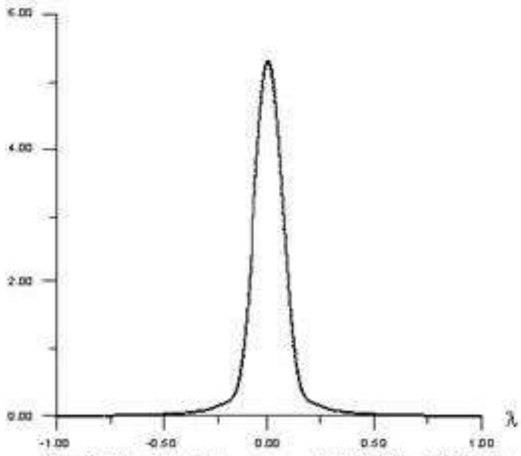

Fig.3. The Function of (2.2.13) with T=50

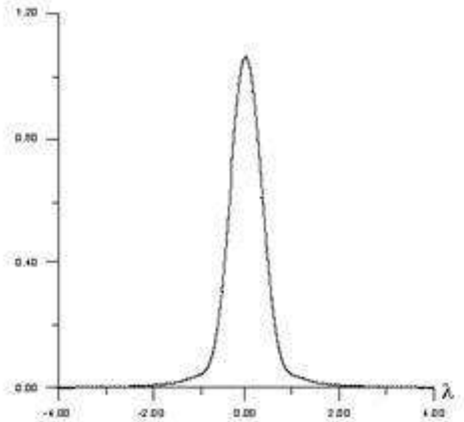

Fig.4. The Function of (2.2.13) with T=10

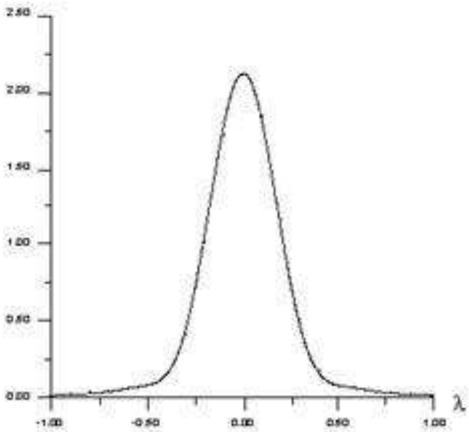

Fig.5. The Function of (2.2.13) with T=20.

Generally speaking, for the symmetry of the function $\Phi^{(T)}(\lambda)$, ($-\infty < \lambda < \infty$) is clear from figures 3, 4 and 5.

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