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Cost Analysis of K-Out Of-N Repairable System Using a Continuous-Time Discrete-State Markov Process

By

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Abstract: Problem statement: This paper deals with a statistical analysis of k-out of-n repairable system with dependent failure and standby support with repair facility. The entire system is working if at least k of its n components are operating. The failure of any working unit of a system results in the reduction of the efficiency of the whole system and as the result, the reliability of the system reduced. The standby unit support increases the reliability of the system. **Approach**: Determine the effect of repairing failed units on the reliability of the system and determine the profit function. Using a continuous-time discrete-state Markov process, the system characteristics are obtained. To validate the theoretical results, numerical computations are derived. Tables and graphs have also been given in the end. **Results**: These results indicated that the system characteristics with repairing a failed unit are greater than the system without repairing with respect to the MTSF, steady state availability and the profit. **Conclusion**: We concluded that the system with repair is better than the system without with respect to the MTSF, steady state availability and the profit.

Keywords: Cost analysis, Repairable system, Dependent failure rates, Markov process, Laplace transform, Mean time to system failure (MTSF), Steady-state availability, Profit function (PF).

1. Introduction.

As long as man has built systems, he has wanted to make them as reliable as possible. Every system has its own characteristics with different failure modes, reliability measures, and analytical methods. The standby redundancy allocation problem has been studied for many different system structures. In a redundant system, some additional paths are created for the proper functioning of the system. If all the redundant parts start working together at the time of operation, then it is termed as parallel redundancy. The redundancy in a system is usually employed to design highly reliable systems. Also, the repairing of failed unit increases the reliability of the system. On the failure of the operating unit, a standby unit is switched on by perfect switching device and failed unit goes for repairing. Thus introducing standby redundant parts and repairing a failed unit may achieve high degree of reliability.

On the other hand, it is assumed that the failure of any unit in k-out of-n system does not affect the functioning of the system. Nevertheless, in practice, the failure of any working unit of a system results in the reduction of the efficiency of the whole system. It increases stress on the others ones and as the result, the failure rate of functional ones increased and the reliability of the system reduced. Thus, dependence occurs and as a result the failure rates of the units degrade.

Many authors have discussed k-out of-n system with dependent failure rates [1, 2]. The system reliability of modeling shared load was investigated by Shao and Lamberson et al., [3]. Mostafa [4] has studied a transient analysis of reliability with and without repair for k-out of-n G systems. Pham [5] has studied availability and Mean Life Time of degraded system with partial repair. Who Kee Chang [6], studied reliability analysis of a repairable parallel system with standby involving human failure and common-cause failures. Madhu Jain, et al., [7] have studied k-out of-n repairable system with dependent failure and standby support. Haggag [8] has studied Cost Analysis of K-Out of-n Repairable System with Dependent Failure and Stand by Support Using Kolmogorov's Forward Equations Method.

Materials and Methods

Many authors have studied k-out of-n repairable system with dependent failure. The question was raised whether the repairing failed units increase the reliability of the system. In this study the statistical analysis of k-out of-n repairable system with dependent failure were discussed to show the system with repair increase the reliability of the system.

We analyze the system by using a continuous-time discrete-state Markov process. After the model is developed a particular case study is discussed to validate the theoretical results. Next, some numerical computations are derived to show the effect of repair and standby support on the system.

2. System Description

The system consists of a main unit, n-subsystems, and s-standbys. The entire system is working if at least k of its main unit and n subsystems are operating. The failure of main unit, which supervises the system, causes the total system failure and has a constant failure rate λ_p. When any of the operating subsystems fails, it is replaced by standby unit and failed unit goes to repair mode with rate μ_i. If all the standbys are consumed, the system works as degraded system until k-subsystems works.

The system is analyzed by using a continuous-time discrete-state Markov process. State i indicating that exactly i subsystems are failed at time 't', (i=1, 2,….., n+ s -k). the failure rate from state i to state i+1 given by Δ_i. The failure of fault coverage is constant and equal to λ_c. The failure rates of all subsystems are constant and same and depend on the number of working units and equal to λ_j, (j=1, 2. …..n). The standby units have constant failure rates β. The system state transition diagram is given in figure 1. The state transition rate of the system is given by: sjms/108

$$\Delta_i = \begin{cases} n\lambda_n - (s-i)\beta & 0 \le i \le s \\ (n+s-i)\lambda_{n+s-i} & s+1 \le i \le n+s-k \end{cases} \quad (1)$$

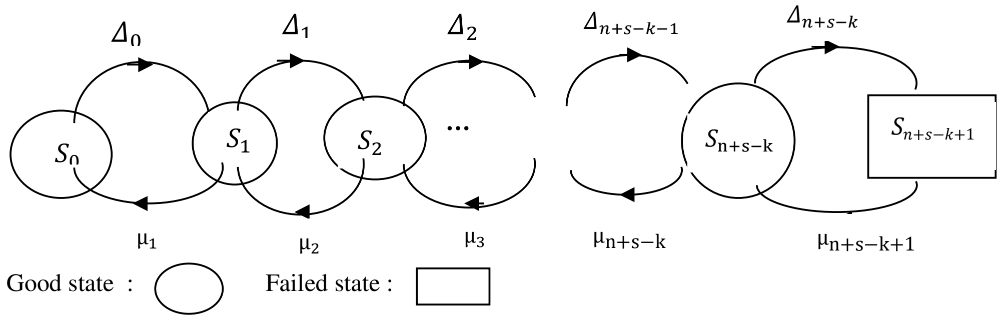

Figure (1): State transition diagram

2.1 Markov Models

Markov model is the tool used in this thesis to analyzing our systems and to describe the performance measures of the operation system such as availability, reliability, mean time to system failure MTSF and cost analysis. The Markov model assumes that the future is independent of the past given the present. Markov Model consists of a list of the possible states S_i of that system, the possible transition paths between those states, and the rate parameters of those transitions. In reliability analysis the transitions usually consist of failures and repairs.

If $P_i(t)$ denoted the probability that the system is in state S_i at time t, t≥0. Therefore the Laplace transform of the probability that the system is in up (operable) state at time 't' is defined by $\overline{P}_{up}(s)$, and by taking inverse Laplace transform, the probability that the system is in upstate at time 't' is given by $P_{up}(t)$. The Busy period is the period where the system states go to repair. Then the busy period due to failure can be formulated as: $B(t) = 1 - P_0(t)$

In the general case Markov Models, both time and space may either be discrete or continuous. In the particular case Markov Models of system reliability evaluation, space is normally represented only as a discrete function, whereas time may either be discrete or continuous. There are two cases of Markov Models: the discrete case, generally known as a Markov chain and the Continuous case, generally known as a Markov Process.

2.1.1 Mathematical model of Markov Process

State-transition equations (difference equations) can be formulated, which result in first-order linear differential equations which can be solved by a lot of ways such as: Laplace Transform and Kolmogorov's equations

2.1.2 Solving the First-Order Linear Differential equations by using Laplace Transform

The advantage of using Laplace transform in the solution of differential equations is that, by transforming from the time domain to s-domain. The problem is reduced from a set of differential equations to a set of simultaneous linear equations which consequently easier to solve.

- **The MTSF**
  To calculate the MTSF we take the limit for the availability ($p_{up}(s)$) of the system.
  $\lim_{s\to 0} p_{up}(s)$

- **The Steady state availability of the system**
  Using Abel's Lemma in Laplace transform, viz
  $\lim_{s\to 0}[s.\bar{F}(s)] = \lim_{t\to\infty} F(t) = F$ (say), we get
  $P_{up} = \lim_{s\to 0} s.\bar{P}_{up}(s)$

- **Reliability of the system**
  First we calculate MTSF of the system, and then we get λ and calculate reliability of the system by:
  $R(t) = e^{-\lambda t},\ \text{where } \lambda = 1/MTSF$

- **The Profit function**
  The expected total profit per unit time incurred to the system in interval (0, t] is given by:
  Profit = total revenue - total cost
  $G(t) = R\mu_{up}(t) - C_i\mu_i(t)$

  Where:

  G (t): is the profit incurred to the system,
  R: is the revenue per unit up-time of the system.
  $\mu_{up}(t)$: the mean up time in interval (0,t].
  $\mu_i(t)$: the expected busy period for repair ,Preventive Maintenance….
  C_i: the service costs per unit of time for repair, Preventive Maintenance …
  and $\mu_i(t) = \int_0^t p_i(t)dt$

- **Steady state Busy period:**
  Busy period is the period where the system states go to repair. In the steady state, the derivatives of the state probabilities become zero and this will enable to compute steady state busy period due to failure as:

  $B(\infty) = \lim_{s\to 0}\left(1 - s\bar{P}_0(s)\right)$

  The expected frequency of preventive maintenance:
  By using the initial condition, then the expected frequency of preventive maintenance per unit time $K(\infty)$is given by

  $K(\infty) = P_i$, where i is the state of Preventive Maintenance

  The Profit function in steady state:

  The expected total profit per unit time incurred to the system in the steady-state is given by:

  Profit = total revenue - total cost
  $PF = RA(\infty) - C_1B(\infty) - C_2K(\infty)$

  Where:

  PF: is the profit incurred to the system,
  R: is the revenue per unit up-time of the system,
  C_1 : is the cost per unit time which the system is under repair

C_2: is the cost per preventive maintenance.

3. Formulation of Mathematical model

By elementary probability and continuity arguments the difference differential equations for the stochastic process of the system which is continuous in time and discrete in space are:

$$\left(\frac{\partial}{\partial t} + \Delta_0 + \lambda_p + \lambda_c\right) P_0(t) = \mu_1 P_1(t) \qquad (3 - 1)$$

$$\left(\frac{\partial}{\partial t} + \Delta_i + \lambda_p + \mu_i\right) P_i(t) = \Delta_{i-1} P_{i-1}(t) + \mu_{i+1} P_{i+1}(t), \quad 1 \le i \le n + s - k \qquad (3 – 2)$$

$$\left(\frac{\partial}{\partial t} + \mu_{n+s-k+1}\right) P_{n+s-k+1}(t) = \Delta_{n+s-k} P_{n+s-k}(t) \qquad (3 – 3)$$

Initial conditions

$P_0(0) = 1$, $P_i(0)$ =0, i=1, 2, 3,..., $n + s - k$ (3 – 4)

2. Solution of the Model

By taking Laplace transform of equations (3-1)-(3-3), and using initial condition (3 – 4), we get:

$$(s + \Delta_0 + \lambda_p + \lambda_c)\bar{P}_0(s) = \mu_1 \bar{P}_1(s) \qquad (4 - 1)$$

$$(s + \Delta_0 + \lambda_p + \mu_i)\bar{P}_i(s) = \Delta_{i-1} P_{i-1}(s) + \mu_{i+1} \bar{P}_{i+1}(s), \ 1 \le i \le n + s - k \qquad (4 – 2)$$

$$(s + \mu_{n+s-k+1})\bar{P}_{n+s-k+1}(s) = \Delta_{n+s-k} \bar{P}_{n+s-k}(s) \qquad (4 – 3)$$

Evaluation of Laplace transform of up and down state Availability

The Laplace transform of the probability that the system is in states S_0, S_i, S_{n+s-k} at time 't' can be evaluated as follows:

$$\bar{P}_0(s) = \frac{1}{s+\Delta_0+\lambda_p+\lambda_c} + \frac{\mu_1}{s+\Delta_0+\lambda_p+\lambda_c} \bar{P}_1(s) \qquad (4 – 4)$$

$$\bar{P}_i(s) = \frac{\Delta_{i-1}}{s+\Delta_i+\lambda_p+\mu_i} \bar{P}_{i-1}(s) + \frac{\mu_{i+1}}{s+\Delta_i+\lambda_p+\mu_i} \bar{P}_{i+1}(s), \quad 1 \le i \le n + s - k \qquad (4 – 5)$$

$$\bar{P}_{n+s-k+1}(s) = \frac{\Delta_{n+s-k}}{s+\mu_{n+s-k+1}} \bar{P}_{n+s-k}(s) \qquad (4 – 6)$$

Therefore the Laplace transform of the probability that the system is in up (operable) and down (failed) state at time 't' are as follows:

$$\bar{P}_{up}(s) = \bar{P}_0(s) + \bar{P}_i(s) \qquad (4 – 7)$$

$$\bar{P}_{down}(s) = 1 - \bar{P}_{up}(s) = \bar{P}_{n+s-k+1}(s) \qquad (4 – 8)$$

3. Particular Case

If we put n=4, k=3, s=2, in equations (4 – 4) - (4 – 6), we have:

$$\bar{P}_0(s) = \frac{1}{s+\Delta_0+\lambda_p+\lambda_c} + \frac{\mu_1}{s+\Delta_0+\lambda_p+\lambda_c} \bar{P}_1(s) \qquad (5 – 1)$$

$$\bar{P}_1(s) = \frac{\Delta_0}{s+\Delta_1+\lambda_p+\mu_1} \bar{P}_0(s) + \frac{\mu_2}{s+\Delta_i+\lambda_p+\mu_1} \bar{P}_2(s) \qquad (5 – 2)$$

$$\bar{P}_2(s) = \frac{\Delta_1}{s+\Delta_2+\lambda_p+\mu_2} \bar{P}_1(s) + \frac{\mu_3}{s+\Delta_2+\lambda_p+\mu_2} \bar{P}_3(s) \qquad (5 – 3)$$

$$\bar{P}_3(s) = \frac{\Delta_2}{s+\Delta_3+\lambda_p+\mu_3} \bar{P}_2(s) + \frac{\mu_4}{s+\mu_4} \bar{P}_4(s) \qquad (5 – 4)$$

$$\bar{P}_4(s) = \frac{\Delta_3}{s+\mu_4} \bar{P}_3(s) \qquad (5 – 5)$$

Evaluation of Laplace transform of up and down state Availability

By solving the above equation recursively, we obtain:

$\overline{P}_0(s) = \langle (s + \Delta_1 + \lambda_p + \mu_1)\{(s + \Delta_2 + \lambda_p + \mu_2)[(s + \Delta_3 + \lambda_p + \mu_3)(s + \mu_4) - \Delta_3\mu_4] - \Delta_2\mu_3(s + \mu_4)\} - \Delta_1\mu_2[(s + \Delta_3 + \lambda_p + \mu_3)(s + \mu_4) - \Delta_3\mu_4]\rangle / A(s)$ (5 – 6)

$\overline{P}_1(s) = \langle \Delta_0\{(s + \Delta_2 + \lambda_p + \mu_2)[(s + \Delta_3 + \lambda_p + \mu_3)(s + \mu_4) - \Delta_3\mu_4] - \Delta_2\mu_3(s + \mu_4)\}\rangle / A(s)$

(5 – 7)

$\overline{P}_2(s) = \langle \Delta_0\Delta_1[(s + \Delta_3 + \lambda_p + \mu_3)(s + \mu_4) - \Delta_3\mu_4]\rangle / A(s)$ (5 – 8)

$\overline{P}_3(s) = \Delta_0\Delta_1\Delta_2(s + \mu_4)/A(s)$ (5 – 9)

$\overline{P}_4(s) = \frac{\Delta_0\Delta_1\Delta_2\Delta_3}{A(s)}$ (5 – 10)

Where

$$A(s) = (s + \Delta_0 + \lambda_p + \lambda_c)\langle (s + \Delta_1 + \lambda_p + \mu_1)\{(s + \Delta_2 + \lambda_p + \mu_2)[(s + \Delta_3 + \lambda_p + \mu_3)(s + \mu_4) - \Delta_3\mu_4] - \Delta_2\mu_3(s + \mu_4)\} - \Delta_1\mu_2[(s + \Delta_3 + \lambda_p + \mu_3)(s + \mu_4) - \Delta_3\mu_4]\rangle - \Delta_0\mu_1\{(s + \Delta_2 + \lambda_p + \mu_2)[(s + \Delta_3 + \lambda_p + \mu_3)(s + \mu_4) - \Delta_3\mu_4] - \Delta_2\mu_3(s + \mu_4)\}$$

Therefore the Laplace transform of the probability that the system is in up (operable) and down (failed) state at time 't' are as follows:

$\bar{P}_{up}(s) = \bar{P}_0(s) + \overline{P}_1(s) + \overline{P}_2(s) + \overline{P}_3(s)$

$\bar{P}_{up}(s) = \langle (s + \Delta_1 + \lambda_p + \mu_1)\{(s + \Delta_2 + \lambda_p + \mu_2)[(s + \Delta_3 + \lambda_p + \mu_3)(s + \mu_4) - \Delta_3\mu_4] - \Delta_2\mu_3(s + \mu_4)\} - \Delta_1\mu_2[(s + \Delta_3 + \lambda_p + \mu_3)(s + \mu_4) - \Delta_3\mu_4] + \Delta_0\{(s + \Delta_2 + \lambda_p + \mu_2)[(s + \Delta_3 + \lambda_p + \mu_3)(s + \mu_4) - \Delta_3\mu_4] - \Delta_2\mu_3(s + \mu_4)\} + \Delta_0\Delta_1[(s + \Delta_3 + \lambda_p + \mu_3)(s + \mu_4) - \Delta_3\mu_4] + \Delta_0\Delta_1\Delta_2(s + \mu_4)\rangle / A(s)$ (5 – 11)

$\bar{P}_{down}(s) = 1 - \bar{P}_{up}(s) = \overline{P}_4(s)$

When there is no repair, the probability that the system is in up (operable) state at time 't' is shown in equation (5 – 12).

$P_{up}(t) = (((s + \Delta_1 + \lambda\_\{p\})(s + \Delta_2 + \lambda\_\{p\})(s + \Delta_3 + \lambda\_\{p\}) + \Delta_0(s + \Delta_2 + \lambda\_\{p\})(s + \Delta_3 + \lambda\_\{p\}) + \Delta_0\Delta_1(s + \Delta_3 + \lambda\_\{p\}) + \Delta_0\Delta_1\Delta_2)/((s + \Delta_0 + \lambda\_\{p\} + \lambda\_\{c\})(s + \Delta_1 + \lambda\_\{p\})(s + \Delta_2 + \lambda\_\{p\})(s + \Delta_3 + \lambda\_\{p\})))$
(5 – 12)

Busy period

From equation (5 – 6), the busy period at time 't' is given by equation (5 – 13):
$B(t) = 1 - P_0(t) = 1 - \langle (s + \Delta_1 + \lambda_p + \mu_1)\{(s + \Delta_2 + \lambda_p + \mu_2)[(s + \Delta_3 + \lambda_p + \mu_3)(s + \mu_4) - \Delta_3\mu_4] - \Delta_2\mu_3(s + \mu_4)\} - \Delta_1\mu_2[(s + \Delta_3 + \lambda_p + \mu_3)(s + \mu_4) - \Delta_3\mu_4]\rangle / A(s)$ (5 – 13)

When there is no repair

$B(t) = 1 - P_0(t) = (1 - 1/(s + \Delta_0 + \lambda_p + \lambda_c))$ (5 – 14)

Mean Time To System Failure (MTSF)

The mean time to system failure (MTSF) for the proposed system can be evaluated as shown in equation (5 – 15).
$MTTF = \lim_{s \to 0} \bar{P}_{up}(s) = \big((\Delta_1 + \lambda_p + \mu_1)\{(\Delta_2 + \lambda_p + \mu_2)[(\Delta_3 + \lambda_p + \mu_3)\mu_4 - \Delta_3\mu_4] - \Delta_2\mu_3\mu_4\} - \Delta_1\mu_2[\mu_4(\Delta_3 + \lambda_p + \mu_3) - \Delta_3\mu_4] + \Delta_0\{(\Delta_2 + \lambda_p + \mu_2)[(\Delta_3 + \lambda_p + \mu_3)\mu_4 - \Delta_3\mu_4] - \Delta_2\mu_3\mu_4\} + \Delta_0\Delta_1[(\Delta_3 + \lambda_p + \mu_3)\mu_4 - \Delta_3\mu_4] + \Delta_0\Delta_1\Delta_2\mu_4\big) / D_1$
(5 - 15)

Where

$D_1 = (\Delta_0 + \lambda_p + \lambda_c)\langle (\Delta_1 + \lambda_p + \mu_1)\{(\Delta_2 + \lambda_p + \mu_2)[\mu_4(\Delta_3 + \lambda_p + \mu_3) - \Delta_3\mu_4] - \Delta_2\mu_3\mu_4\} - \Delta_1\mu_2[\mu_4(\Delta_3 + \lambda_p + \mu_3) - \Delta_3\mu_4]\rangle - \Delta_0\mu_1\{(\Delta_2 + \lambda_p + \mu_2)[\mu_4(\Delta_3 + \lambda_p + \mu_3) - \Delta_3\mu_4] - \Delta_2\mu_3\mu_4\}$

When there is no repair, the mean time to system failure (MTSF) is shown in equation (5 – 16).

$$MTTF = (((\varDelta_1+\lambda_p)(\varDelta_2+\lambda_p)(\varDelta_3+\lambda_p)+\varDelta_0(\varDelta_2+\lambda_p)(\varDelta_3+\lambda_p)+\varDelta_0\varDelta_1(\varDelta_3+\lambda_p)+\varDelta_0\varDelta_1\varDelta_2)/((\varDelta_0+\lambda_p+\lambda\_\{c\})(\varDelta_1+\lambda_p)(\varDelta_2+\lambda_p)(\varDelta_3+\lambda_p))) \qquad (5-16)$$

Steady state availability of the system

Using Abel's Lemma in Laplace transform, we get the steady state availability of the system as shown in equation (5 – 17).

$$A(\infty)=P_{up}=\lim_{s\to 0} s\bar{P}_{up}(s)=\Big((\varDelta_1+\lambda_p+\mu_1)\{(\varDelta_2+\lambda_p+\mu_2)[(\varDelta_3+\lambda_p+\mu_3)(\mu_4)-\varDelta_3\mu_4]-\varDelta_2\mu_3\mu_4\}-\varDelta_1\mu_2[\mu_4(\varDelta_3+\lambda_p+\mu_3)-\varDelta_3\mu_4]+\varDelta_0\{(\varDelta_2+\lambda_p+\mu_2)[(\varDelta_3+\lambda_p+\mu_3)\mu_4-\varDelta_3\mu_4]-\varDelta_2\mu_3\mu_4\}+\varDelta_0\varDelta_1[(\varDelta_3+\lambda_p+\mu_3)\mu_4-\varDelta_3\mu_4]+\varDelta_0\varDelta_1\varDelta_2\mu_4\Big)/D$$
(5 – 17)

Where

$$D=(\Delta_0+\lambda_p+\lambda_c)((\Delta_2+\mu_2+\lambda_p)(\mu_4(\Delta_3+\mu_3+\lambda_p)-\Delta_3\mu_4)-\Delta_1\mu_2(\mu_4+\Delta_3+\mu_3+\lambda_p)-\Delta_2\mu_3\mu_4+(\Delta_1+\mu_1+\lambda_p)(\mu_4(\Delta_3+\mu_3+\lambda_p)-\Delta_3\mu_4-\Delta_2\mu_3+(\Delta_2+\mu_2+\lambda_p)(\mu_4+\Delta_3+\mu_3+\lambda_p)))+(\Delta_1+\mu_1+\lambda_p)((\Delta_2+\mu_2+\lambda_p)(\mu_4(\Delta_3+\mu_3+\lambda_p)-\Delta_3\mu_4)-\mu_4\Delta_2\mu_3)-\Delta_1\mu_2(\mu_4(\Delta_3+\mu_3+\lambda\_\{p\})-\Delta_3\mu_4)-\Delta_0\mu_1(\mu_4(\Delta_3+\mu_3+\lambda_p)-\Delta_3\mu_4-\Delta_2\mu_3+(\Delta_2+\mu_2+\lambda\_\{p\})(\mu_4+\Delta_3+\mu_3+\lambda_p))$$

When there is no repair the steady state availability of the system as shown in equation (5 – 18).

$$A(\infty)=(((\varDelta_1+\lambda\_\{p\})(\varDelta_2+\lambda\_\{p\})(\varDelta_3+\lambda\_\{p\})+\varDelta_0(\varDelta_2+\lambda\_\{p\})(\varDelta_3+\lambda\_\{p\})+\varDelta_0\varDelta_1(\varDelta_3+\lambda\_\{p\})+\varDelta_0\varDelta_1\varDelta_2)/D_2$$
(5 – 18)

Where

$$D_2=((\varDelta_1+\lambda\_\{p\})(\varDelta_2+\lambda_p)(\varDelta_3+\lambda_p)+(\varDelta_1+\lambda_p)(\varDelta_2+\lambda_p)(\varDelta_0+\lambda_c+\lambda_p)+(\varDelta_1+\lambda_p)(\varDelta_3+\lambda_p)(\varDelta_0+\lambda_c+\lambda_p)+(\varDelta_2+\lambda_p)(\varDelta_3+\lambda_p)(\varDelta_0+\lambda\_\{c\}+\lambda_p)))$$

Steady state busy period

The steady state busy period B(∞) is given by equation (5 – 19).

$$B(\infty)=\lim_{s\to 0}\big(1-s\bar{P}_0(s)\big)=1-\frac{N_1}{D} \qquad (5-19)$$

Where: $N_1=(\varDelta_1+\lambda_p+\mu_1)\{(\varDelta_2+\lambda_p+\mu_2)[(\varDelta_3+\lambda_p+\mu_3)\mu_4-\varDelta_3\mu_4]-\varDelta_2\mu_3\mu_4\}-\varDelta_1\mu_2[(\varDelta_3+\lambda_p+\mu_3)\mu_4-\varDelta_3\mu_4]$

When no repair

$$B(\infty)=\Big(1-\lim_{s\to 0} s\bar{P}_0(s)\Big)=1-\frac{1}{(\Delta_0+\lambda_p+\lambda_c)} \qquad (5-20)$$

Cost analysis:

The expected total profit per unit time incurred to the system is given by equation (5 – 21):
Profit = total revenue - total cost
$\mathrm{PF}=\mathrm{R}\times\mathrm{EOT(t)}-\mathrm{C}\times\mathrm{EBP(t)}$ (5 – 21)

The expected total profit per unit time incurred to the system in the steady-state is given by equation (5 – 21).
Profit = total revenue - total cost
$\mathrm{PF}=\mathrm{R}\times\mathrm{A}(\infty)-\mathrm{C}\times\mathrm{B}(\infty)$ (5 – 22)
Where,
PF: is the profit incurred to the system,
R: is the revenue per unit up-time of the system,
C: is the cost per unit time which the system is under repair.

1. Results

a- Availability analysis

If we put: $\lambda_p=.002, \lambda_c=.003, \beta=.007, \lambda_4=.01, \lambda_3=.03., \lambda_2=.05, \mu_1=.5, \mu_2=.4, \mu_3=.3, \mu_4=.2$
(6 – 1)

in equation (5 - 11), and by taking inverse Laplace transform, one may get the probability that the system is in upstate at time 't'.

$P_{up}(t) = 1.0091exp\{(-4.6674 \times 10^{-3}t)\} + 7.1076 \times 10^{-4}exp\{(-0.12175t)\} - 0.01663exp\{(-0.3263t)\} - 5.1542 \times 10^{-3}exp\{(-0.502475t)\} + 1.1979 \times 10^{-2}exp\{(-0.68682t)\}$
(6 – 2)

When there is no repair

$P_{up}(t) = 38.39exp\{(-0.042t)\} - 1.4309exp\{(-0.092t)\} - 64.585exp\{(-0.049t)\} + 28.626exp\{(-0.059t)\}$.
(6 – 3)

Setting $t = 0, 1, 2, \dots\dots\dots$, in equation (6 – 2), (6 – 3), one can compute table 1. Variation of availability w.r.t. time is shown in Figure 2.

Time (t)	Availability with repair	Availability without repair
0	1.0000	1.0000
10	0.96264	0.95548
20	0.9192	0.90281
30	0.87727	0.82513
40	0.83725	0.72434
50	0.79907	0.61172
60	0.76263	0.49931
70	0.72785	0.39591
80	0.69466	0.30635
90	0.66298	0.23223
100	0.63275	0.17302

Table (1) Variation of availability with time 't'

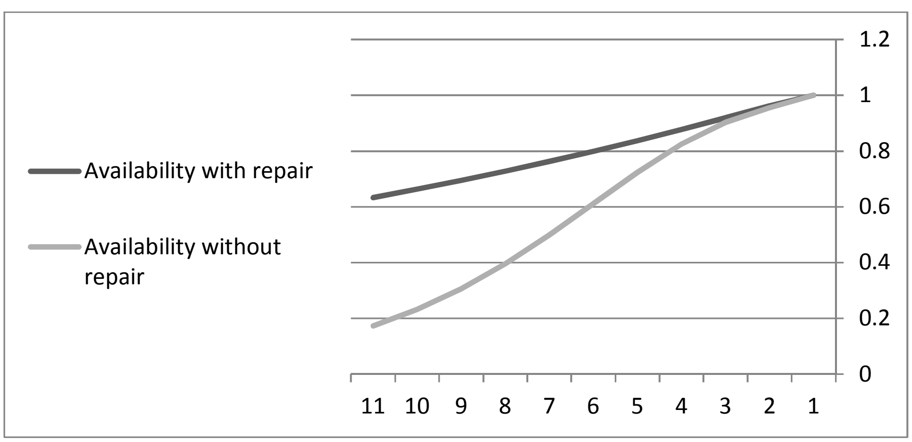

Figure (2): availability w.r.t. time with and without repair.

a- Expected Operational Time (EOT)

From equation (6 – 2), one may get:

$$EOT(t) = -216.2(\exp\{(-4.6674 \times 10^{-3}t)\} - 1) - 5.8379 \times 10^{-3}(\exp\{(-0.12175t)\} - 1) + 5.0965 \times 10^{-2}(\exp\{(-0.3263t)\} - 1) + 1.0258 \times 10^{-2}(\exp\{(-0.502475t)\} - 1) - 1.7441 \times 10^{-2}(\exp\{(-0.68682t)\} - 1)$$
(6 – 4)

When there is no repair

From equation (6 – 3), one may get:

$$EOP(t) = 914.05(\exp\{-0.042t\} - 1) + 15.553(\exp\{-0.092t\} - 1) + 1318.1(\exp\{-0.049t\} - 1) - 485.19(\exp\{-0.059t\} - 1)$$
(6 – 5)

Setting $t = 10, 20, 30, \ldots \ldots \ldots$, in equation (6 – 4), (6 – 5), one can compute table (2). Variation of expected operational time w.r.t. time is shown in Figure (3).

Time (t)	Expected Operational Time (EOT) with repair	Expected Operational Time (EOT) without repair
10	9.8210	9.7603
20	19.230	19.061
30	28.211	27.718
40	36.782	35.478
50	44.962	42.161
60	52.769	47.711
70	60.220	52.176
80	67.331	55.674
90	74.118	58.354
100	80.595	60.368

Table (2): Variation of expected operational time w.r.t. time with and without repair.

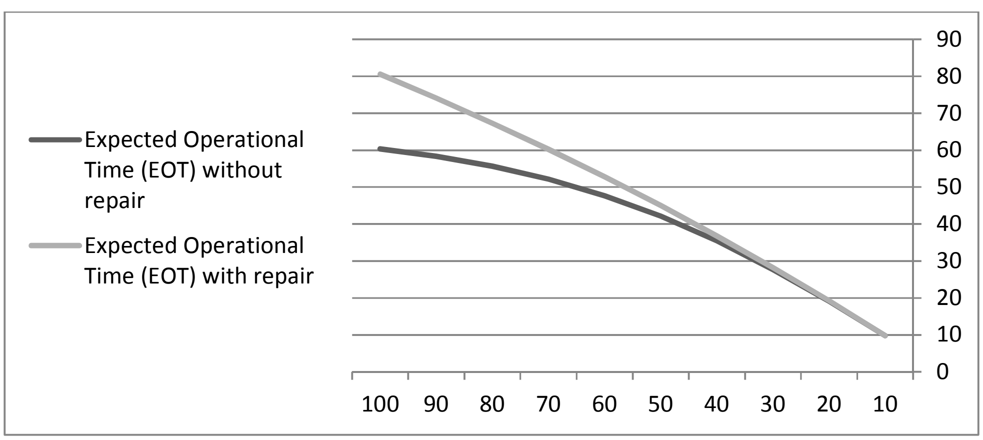

Figure (3): Variation of expected operational time w.r.t. time with and without repair.

a- Busy period

If we put: $\lambda_p = .002, \lambda_c = .003, \beta = .007, \lambda_4 = .01, \lambda_3 = .03., \lambda_2 = .05, \mu_1 = .5, \mu_2 = .4, \mu_3 = .3, \mu_4 = .2$, in equation (5 – 13), we get

$$B(t) = 0.58376exp(-0.50248t) - 0.3609e^{-0.3263t} - 0.31207exp(-0.68682t) - 0.74114exp(-4.6674 \times 10^{-2}t) - 0.16966exp(-0.12175t) \quad (6-6)$$

When there is no repair, from equation (5 – 14)

$$B(t) = -exp\{-0.059t\} \quad (6-7)$$

Expected Busy Period (EBP)

From equation (6 –), one may get:

$$\text{EBP}(t) = (-1.1618(exp(-0.50248 * t) - 1) + 1.106exp\{-0.3263t\} + 0.45437(exp(-0.68682t) - 1) + 15.879(exp(-.04\,6674t) - 1) + 1.3935(exp(-0.12175t) - 1)) \quad (6-8)$$

When there is no repair

$$\text{EBP(t)} = 16.949(\exp\{-0.059\text{t}\} - 1\,) \quad (6-9)$$

a- Cost Analysis

From equation (6 – 4), (6 – 8), one may get:

$$\text{PF(t)} = \text{R} \times \text{EOT(t)} - \text{C} \times \text{EBP(t)} = \text{R}\{-216.2(\exp\{(-4.6674 \times 10^{-3}\text{t})\} - 1) - 5.8379 \times 10^{-3}(\exp\{(-0.12175\text{t})\} - 1) + 5.0965 \times 10^{-2}(\exp\{(-0.3263\text{t})\} - 1) + 1.0258 \times 10^{-2}(\exp\{(-0.502475\text{t})\} - 1) - 1.7441 \times 10^{-2}(\exp\{(-0.68682\text{t})\} - 1)\} - \text{C}(-1.1618(exp(-0.50248t) - 1) + 1.106exp\{-0.3263t\} + 0.45437(exp(-0.68682t) - 1) + 15.879(exp(-.04\,6674t) - 1) + 1.3935(exp(-0.12175t) - 1)) \quad (6-10)$$

When there is no repair: From equation (6 – 5), (6 – 9), one may get:

$$\text{PF(t)} = \text{R} \times \text{EOT(t)} - \text{C} \times \text{EBP(t)} = \text{R}\{-914.05(\exp\{-0.042\text{t}\} - 1) + 15.553(\exp\{-0.092\text{t}\} - 1) + 1318.1(\exp\{-0.049\text{t}\} - 1) - 485.19(\exp\{-0.059\text{t}\} - 1)\} - \text{C}(16.949(\exp\{-0.059\text{t}\} - 1\,))$$

(6 – 11)

Setting values for the cost coefficient C and revenue R, one can get the expected total gain in the interval (0, t] for different values of 't'. By using equation (6 – 1) in equation(6 – 10), (6 – 11) and by setting $t = 0, 1, 2, 3, \dots \dots$, for $R = 1000, C = 100$, one can compute table (4). Figure (5) shows variation of expected total gain w.r.t. time.

Time (t)	PF(t) with repair	PF(t) without repair
0	0.0	0.0
10	10437.	10516
20	20250	20235
30	29472	29125
40	38192	37013
50	46464.	43767
60	54329	49356
70	61816	53844
80	68950	57354

90	75751	60040
100	82237	62058

Table (3) : Variation of expected total gain w.r.t. time with and without repair.

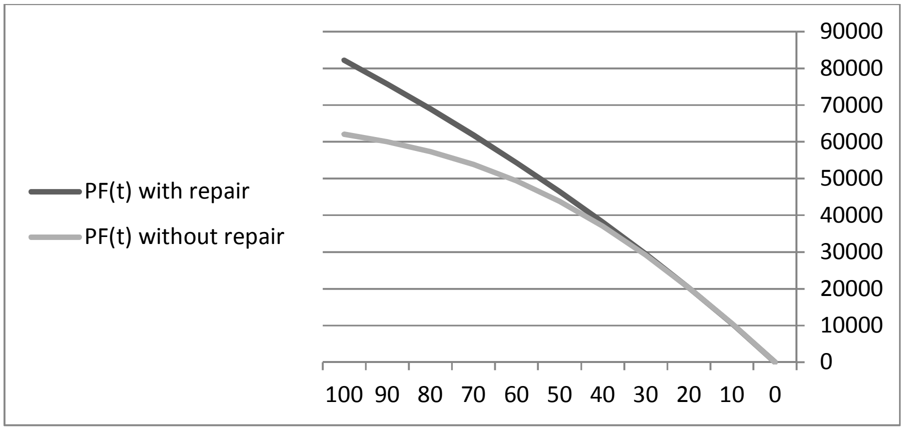

Figure (4) Variation of expected total gain w.r.t. time with and without repair

a- MTSF Analysis

By using equation (6 – 1) in equation (5 – 15), (5 – 16), and by setting $\lambda_4 = .01, .02, .03, \dots\dots$, one can compute table (5). Variation of MTSF for different values of λ_4 is shown in Figure (6).

λ_4	MTSF with repair	MTSF without repair
.01	205.83	65.618
.02	199.12	41.931
.03	185.24	32.594
.04	166.33	27.580
.05	144.52	24.448
.06	121.83	22.306
.07	99.934	20.748
.08	80.100	19.564
.09	63.163	18.633
.1	49.550	17.882

Table (4): Variation of MTSF for different values of λ_4 with and without repair.

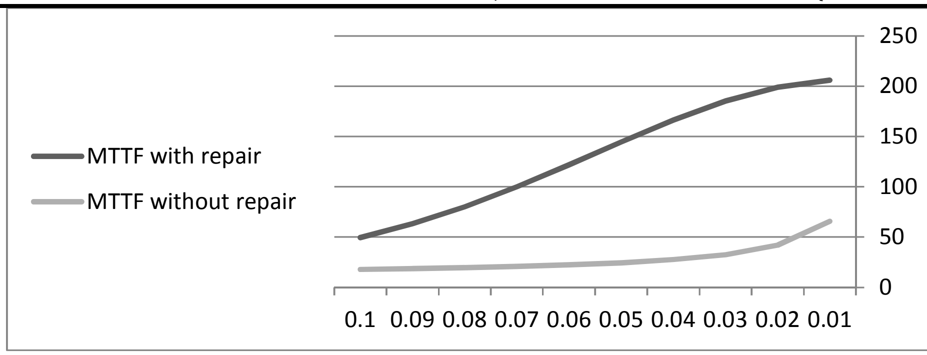

Figure (5) Variation of MTSF for different values of λ_4 with and without repair.

By using equation (6 – 1) in equation (5 – 15), (5 – 16), and by setting $\lambda_1 = .01, .02, .03, \dots \dots,$ one can compute table (6). Variation of MTSF for different values of λ_p is shown in Figure (7)

λ_p	MTSF with repair	MTSF without repair
.001	272.21	68.647
.002	213.93	65.618
.003	176.21	62.803
.004	149.79	60.182
.005	130.26	57.738
.006	115.24	55.455
.007	103.33	53.318
.008	93.642	51.317
.009	85.619	49.438
.01	78.862	47.673

Table (5): Variation of MTSF for different values of λ_p with and without repair.

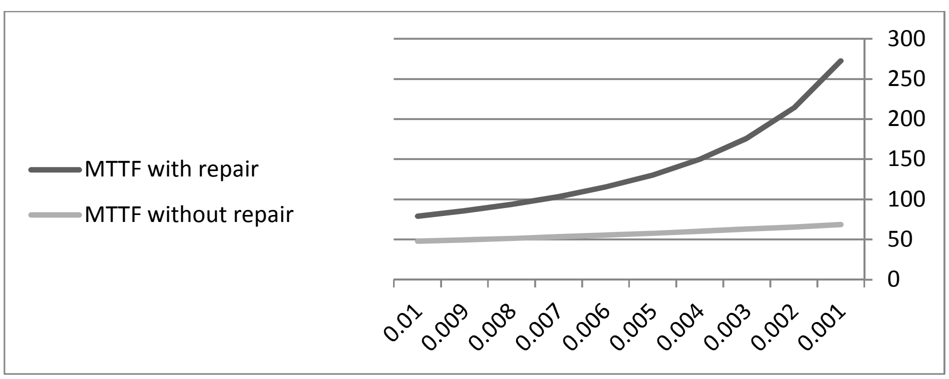

Figure (6) Variation of MTSF for different values of λ_p with and without repair.

a- Steady-state Availability

By using equation (6 – 1) in equation (5 – 17), (5 – 18), and by setting $\lambda_p = .001, .002, \dots \dots,$ one can compute table(7). Variation of steady-state availability for different values of λ_p is shown in Figure (8).

λ_p	Steady state availability With repair	Steady state availability Without repair
.001	0.94723	0.93455
.002	0.93429	0.91090
.003	0.92175	0.88862
.004	0.9096	0.86760
.005	0.89782	0.84774
.006	0.88639	0.82896
.007	0.87529	0.81118
.008	0.86451	0.79433
.009	0.85404	0.77834
.01	0.84386	0.76315

Table (6): Variation of availability for different values of λ_p with and without repair.

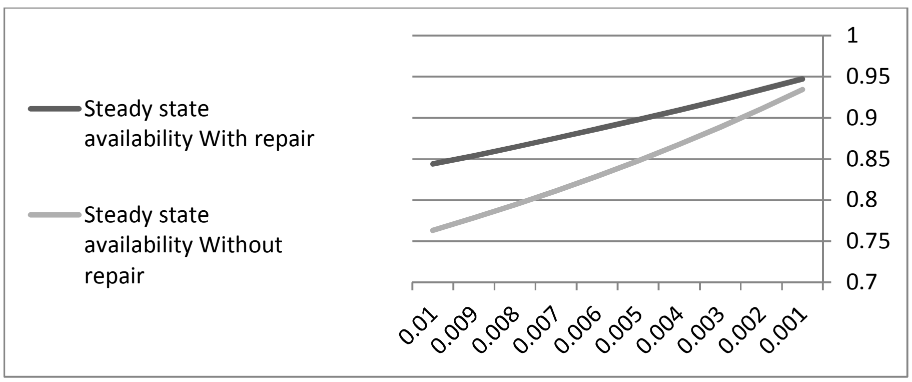

Figure (7) Variation of steady-state availability for different values of λ_p with and without repair.

By using equation (6 – 1) in equation (5 – 19), (5 – 20), and by setting $\lambda_p = .001, .002, \dots\dots,$ one can compute table(8). Variation of steady-state busy period for different values of λ_p is shown in Figure(9).

λ_p	Steady state busy period With repair	Steady state busy period Without repair
.001	0.15581	0.0
.002	0.16711	0.0
.003	0.17805	0.0
.004	0.18865	0.0
.005	0.19894	0.0
.006	0.20892	0.0

.007	0.21861	0.0
.008	0.22802	0.0
.009	0.23716	0.0
.01	0.24605	0.0

Table (7): Variation of availability for different values of λ_p

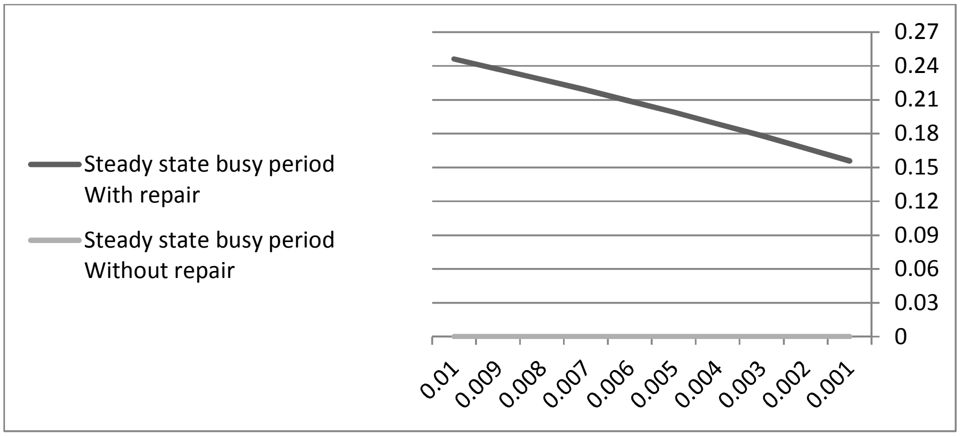

Figure (8) Variation of steady-state availability for different values of λ_p with and without repair.

Also, by using equation (6 – 1) in equation (5 – 21), (5 – 22), and by setting $\lambda_p = .001, .002, \dots\dots,$ for $R = 1000, C = 100$, one can compute table (9). Figure (10) show expected total gain increase for different values of λ_p.

λ_p	PF with repair	PF without repair
.001	931.65	934.55
.002	917.58	910.90
.003	903.95	888.62
.004	890.74	867.60
.005	877.93	847.74
.006	865.50	828.96
.007	853.43	811.18

.008	841.71	794.33
.009	830.32	778.34
.01	819.26	763.15

Table (8): expected total profit in the steady state for different values of λ_p with and without repair.

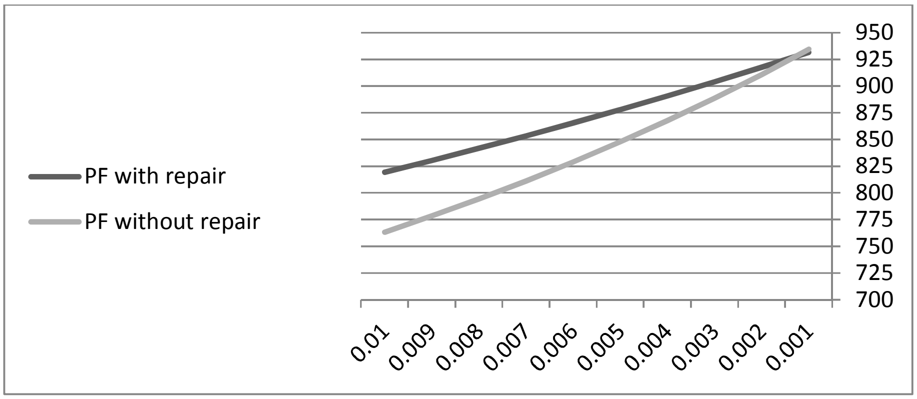

Figure (9) show expected total gain decreases for different values of λ_p

1. Interpretation of the results

From figure (2) we observing that the availability decreases w.r.t., time and from figure (4) we observing that the profit increases w.r.t., time. From figure (5, 6) by comparing the MTSF with respect to λ_p, λ_4 theoretically and graphically. It was observing that:

The increase of failure rates (λ_p, λ_4) at constant $, \lambda_c = .003, \beta = .007, \lambda_3 = .03., \lambda_2 = .05, \mu_1 = .5, \mu_2 = .4, \mu_3 = .3, \mu_4 = .2$, R =1000, C= 100, the MTSF of the system decreases for both system with and without repair, but the system with repair is greater than the system without repair. We conclude that: the MTSF of the system with repair is better than the system without repair.

Also from figure (7, 9) by comparing the steady state availability and the steady state profit with respect to λ_p theoretically and graphically. It was observing that:

The increase of failure rate (λ_p) at constant $, \lambda_c = .003, \beta = .007, \lambda_3 = .03., \lambda_2 = .05, \mu_1 = .5, \mu_2 = .4, \mu_3 = .3, \mu_4 = .2$, R =1000, C= 100, the steady state availability and the steady state profit of the system decreases for both system with and without repair, but the system with repair is greater than the system without repair. We conclude also that: the steady state availability and the steady state profit of the system with repair is better than the system without repair.

From figure (3, 8), by comparing the expected operational time with respect to failure rate λ_p theoretically and graphically. It was observing that: The increase of failure rate λ_p at constant $\lambda_c = .003, \beta = .007, \lambda_4 = .01, \lambda_3 = .03., \lambda_2 = .01, \mu_1 =. .02, \mu_2 = .03, \mu_3 = .03, \mu_4$ =.06, R =1000, C= 100, the expected operational time increases for both systems with and without repair but with repair are greater than without repair.

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